The Simpler, the Better: A New Challenge for Fair-Division Theory

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Abstract

According to John Rawls, there exists a *perfect* procedural justice for which there is no conflict between process and outcome. One such procedure is the *Divide and Choose*. Recently, the mathematical theory of fair-division extended this idea by developing procedures that offer fairer outcomes and a better guarantee of justice. Here, we tested the extent to which the distributive and procedural properties of these *perfect* and improved division procedures were perceived as more satisfactory and fairer than imperfect division procedures. Thirty-nine pairs of participants divided six $10 gift certificates between them using seven division procedures. They rated their satisfaction and their perceived fairness before and after they executed each division procedure. Contrarily to our hypothesis, the results show that perfect procedural justice does not really translate into the perception of a fairer and more satisfactory outcome and process. The most sophisticated division procedures failed to select fair and satisfactory solutions.

Keywords: Fair-division theory, fair-division procedures, satisfaction, perceived fairness, procedural and distributive justice, John Rawls.

Introduction

Social psychological studies on justice distinguish between the perception of the outcome and the perception of the process that leads to the outcome (Lind & Tyler, 1988). The former concerns issues of *distributive justice* such as the criteria under which an outcome is considered fair or unfair (e.g., equitability or envy-freeness), whereas the latter relates to issues of *procedural justice* such as voicing—to express oneself in the process of justice. Perception of distributive and procedural justice does not always coincide: for instance, a procedure seen as fair can lead to undeserved outcomes (e.g., an innocent individual found guilty by a court of justice) and conversely, an outcome perceived as fair can result from an unjust procedure (e.g., a monarch applying a just sentence).

According to Rawls (1971), there exists a perfect procedural justice for which there is no conflict between process and outcome. This idea is based on an age-old fair-division procedure termed Divide and Choose. A first player divides a 'cake' (or any divisible object) in what she considers to be two equal pieces, and a second player chooses the piece she sees as the largest. This procedure exemplifies perfect procedural justice because it has an independent criterion for what constitutes a fair outcome and a process that *guarantees* that such an outcome will be reached. More specifically, the solution derived from Divide and Choose is *envy-free* since both players will have no incentive to exchange their share with the other player's share. Also, its process always leads to such solution given that the players comply with the rules and adhere to specific mathematical assumptions. Nonetheless, Divide and Choose has weaknesses. First, it only applies to conflicts that involve two parties and a divisible good (e.g., money). Second, it is vulnerable to strategic manipulation (Crawford & Heller, 1979). Third, it does not guarantee that the divider will "cut the cake" in the most efficient way. A division is *efficient* if no other division can make one participant better off without hurting another.

The mathematical theory of fair-division extended Rawls' idea of a perfect procedural justice mainly by focusing on solutions that are fair *and* efficient, and by strengthening the guarantee of fairness with game-theoretic tools. In the context of game theory, a procedure's fair and efficient solution is said to be guaranteed when the strategy it prescribes (e.g., "cut the cake in what you consider two equal pieces") is optimal for rational and self-regarding players.

Following these improvements, mathematicians have recently designed dozens of fair-division procedures presenting sophisticated mechanisms (for a review, see Barbanel, 2004; Brams, 2008; Brams & Taylor, 1996a; Moulin, 2003; Robertson & Webb, 1998; Young, 1994). One of such algorithms, called the Adjusted Winner (Brams & Taylor, 1996a), has recently been patented in the United

States by New York University (U.S. Patent 5983205). This procedure guarantees an envy-free, equitable and efficient solution (a brief description of the procedure is provided at viscog.psy.umontreal.ca/~nicolas/FDAs.pdf). It has been shown to be useful to resolve bipartite conflicts involving indivisible goods such as divorces (Brams & Taylor, 1996b) and international disputes (Denoon & Brams, 1997; Massoud, 2000).

Despite their immense potential for conflict resolution, very few empirical studies have put fair-division procedures to the test (Daniel & Parco, 2005; Dupuis-Roy & Gosselin, 2009, submitted; Pratt & Zeckhauser, 1990; Schneider & Krämer, 2004). Therefore, many key issues regarding the distributive and procedural determinants of justice involved in the implementation of fair-division procedures with humans have yet to be investigated. The following experiment examines how Rawls' theoretical notion of procedural perfection translates into the perception of a fair and satisfactory process and outcome.

Table 1. List of the main properties of seven fair-division procedures.

Name	**Criteria of fairness**	**Guarantee of fairness**
Strict Alternation	None	None
Balanced Alternation	None	None
Divide and Choose	Envy-freeness	Weak
Adjusted Knaster	Envy-freeness Equitability Efficiency	Strong
Adjusted Winner	Envy-freeness Equitability Efficiency	Strong
Compensation Procedure	Envy-freeness Efficiency	Strong
Price Procedure	Envy-freeness Efficiency	Strong

Hypotheses

Based on Rawls' theoretical conception of procedural justice, we expect fair-division procedures to produce more satisfactory outcomes and elicit more confidence in the process of justice than *imperfect* procedures, that is, procedures which do not implement any criterion of fairness or do not offer any guarantee of reaching such a criterion (Hypothesis 1). To test this hypothesis, a selection of fair-division procedures will be compared with two imperfect procedures: the Strict Alternation and the Balanced Alternation. In the Strict Alternation, one participant from a pair of participants, say P1, is randomly chosen to select a first object; then the other participant, say P2, selects another object; previous steps are repeated until all objects are chosen. The Balanced Alternation is simply an improved version of the Strict Alternation where the selection cycle is balanced: when there are four objects, the selection cycle becomes P1-P2-P2-P1 rather than P1-P2-P1-P2. These procedures do not meet any criterion of fairness *per se* and are thus not considered mathematically fair (Brams & Taylor, 1996). Nonetheless, the Strict Alternation is still used nowadays by juridical institutions to divide inheritances or family patrimonies.

Considering the recent improvements brought by fair-division theory, one can also expect more sophisticated fair-division procedures such as the Adjusted Winner to perform better than more simple and intuitive ones such as the Divide and Choose (Hypothesis 2). This hypothesis will be tested by contrasting the perception toward distributive and procedural properties of four sophisticated fair-division procedures (the Adjusted Knaster by Raith, 2000; the Adjusted Winner by Brams & Taylor, 1996a; the Compensation Procedure by Haake, Raith & Su, 2000; the Price Procedure by Pratt, 2007) with a simple one (the Divide and Choose). Note that all these division procedures are described at viscog.psy.umontreal.ca/~nicolas/FDAs.pdf.

As shown in Table 1, the four most sophisticated fair-division procedures that were selected for this experiment all guarantee at least an envy-free and efficient outcome (column 2). The Adjusted Knaster and the Adjusted Winner also guarantee *equitability.* A solution is equitable when all players put the same value on their own share. The guarantee of fairness of these procedures is considered strong because the strategies they prescribe also lead to an optimal outcome for rational and self-regarding participants (column 3). Since the Divide and Choose is vulnerable to strategic manipulation (Crawford & Heller, 1979), its guarantee of fairness is considered weak.

Experimental Design

To disentangle the distributive and procedural properties of a fair-division procedure, the subject's perception of a division solution will be measured before and after she applies a given procedure. In this context, a change of perception can be readily attributed to the procedural properties since the distributive properties remain constant. The perception of the procedural and distributive properties will be assessed by a direct subjective measure of perceived fairness and satisfaction. If the former specifically elicits judgments based on interpersonal comparisons and social values, the latter could integrate everything that influences one's well-being—including fairness.

The seven division procedures will be tested in the context of the division of six $10 gift certificates between two participants. First, the participants will rate their satisfaction and perceived fairness regarding the 64 possible discrete division solutions; then, they will learn and execute each division procedure in a random order; and finally, they will rate their satisfaction and perceived fairness regarding the division solution provided by each procedure.

Methods

Participants

Thirty-nine dyads of students from the Université de Montréal (mostly from the Departments of Psychology and Mathematics) completed a three-hour lab experiment. Twenty dyads were friends (20 men and 20 women) who had known each other for at least three years (M=8.37 years; SD=4.73 years) and the remaining dyads were strangers (20 men and 18 women) that were randomly paired. All participants were naïve to the purpose of the experiment.

Indivisible Goods

Before the experiment, the participants were shown a list of 70 vendors (e.g., Esso, Canadian Tire, Starbucks) and had to select at least 15 vendors from which they would have liked to receive a $10 gift certificate. After the participants were paired in dyad, six vendors that were in both participants' selections were selected. Color pictures of these vendors were taken from the Internet and resized to span approximately the same surface of the computer monitor.

Apparatus

The experiment was conducted on six computers (PC and Macintosh) that were linked to a server through a local area network. Experimental programs displaying the instructions (or questions) and recording the answers were written for the Matlab environment.

Experimental Procedure

On their arrival in the lab, all dyads were brought into a large computer room and were seated opposite to each other. Those in the "stranger" group were told that they would be randomly paired with a stranger who was conducting the experiment in another experimental room. After being instructed to remain silent, they were told that the goal of the experiment was to divide six $10 gift certificates between them and their partner with the help of seven division procedures that they would have to learn and apply. They were advised to pay close attention to the instructions even if the local computer would perform all mathematical computations required by the procedure. They were asked to engage in this experiment as if they were part of a problematic family succession in which they had to split the six gift certificates. Finally, they were told that the division solutions they would obtain from the division procedures would determine which gift certificates they would receive as a compensation for their participation.

The experiment lasted approximately three hours and involved three steps. (1) Participants first answered some questions regarding their socioeconomic status, their level of education and their level of friendship with their partner. Then, they were shown images of the six disputed certificates and were asked to express their preferences for every item by distributing a total of 100 points over them (the more points attributed to an item, the greater the preference). (2) Next, the participants rated their satisfaction and their perceived fairness toward the 64 possible discrete ways of dividing six items between two persons. For each rating, a division solution was displayed on the monitor screen until a response was given: the top row contained the items given to the participant and the bottom row contained the items given to his/her partner. Participants were told to use a scroll bar to express their level of satisfaction or fairness associated with the illustrated division solution on a scale ranging from 0 ('not satisfied at all' or 'totally unfair') to 100 ('fully satisfied' or 'totally fair'). Note that the division solutions were presented in a random order. (3) Participants read a brief description of a given division procedure and answered three questions assessing their comprehension. After having correctly answered all questions, they were submitted to five practice trials during which they applied the procedure with a fictitious partner (the computer) whose preferences for the six items were public. Then, they applied the fair-division procedure with their partner and evaluated their satisfaction and perceived fairness toward its division solution. They repeated this process seven times, one time per procedure. The order in which each dyad applied the procedures was randomized.

Results

Figure 1 shows the average satisfaction (upper plot) and perceived fairness (lower plot) measured before (left side) and after (right side) the implementation of the fair-division procedures (x-axis). At first glance, these group results contradict our hypotheses: the solutions produced by the most sophisticated fair-division procedures are perceived as less fair and less satisfactory than the ones derived from the imperfect and simple procedures. Even more surprisingly, the Strict Alternation produced, on average, the fairest solutions of all. In order to better understand these seemingly counterintuitive results, the procedural and distributive factors will be disentangled with analyses of variance.

First, a MANOVA was run to evaluate the differences between both assessment of the dependent variables (TIME) and possible interactions with the type of fair-division procedures (PROC). In addition to these three within-subjects factors (dependent variables (DVs), TIME and PROC), we also examined the differences between the types of dyad (either consisting of friends or strangers). Wilks test revealed no difference between types of dyad ($F(1,81)=0.01$, ns) but significant interactions between PROC and TIME ($F(6,81)=19.81$, $p<0.001$), PROC and

DVs ($F(6,81)=40.92$, $p<0.001$), TIME and DVs ($F(1,86)=6.29$, $p<0.02$), and PROC, TIME and DVs ($F(6,81)=13,49$, $p<0.001$). To boost the power of the analysis, types of dyad were pooled together, and a PROC x TIME within-subject ANOVA was performed on each DV separately (type I error was controlled with Bonferroni adjustments).

The first ANOVA was computed on satisfaction ratings. Results show a main effect of PROC ($F(6,82)=9.53$, $p<0.001$) and TIME ($F(1,87)=42.05$, $p<0.001$) but no interaction between them ($F(6,82)=0.81$, *ns*). This means that the application of a fair-division procedure, regardless of its type, increased satisfaction. It also suggests that the differences between the procedures do not stem from their implementation *per se* but from the type of division solution they produced. In other words, procedures differ from one another because of their distributive, not their procedural properties.

A similar ANOVA was computed on perceived fairness. Results show two significant main effects and a significant interaction between PROC and TIME ($F(6,82)=8.10$, $p<0.001$). This means that the procedural properties of some division procedures had an effect on perceived fairness[1]. Statistical contrasts were computed at each level of PROC: we found that the Adjusted Knaster ($F(1,87)=46.41$, $p<0.001$) and the Compensation Procedure ($F(1,87)=16.91$, $p<0.001$) had a significant positive effect on the perceived fairness. This indicates that even though imperfect and simple procedures produced the fairest solutions, their procedural properties had no positive effect on the perception of fairness. This is also true for the Adjusted Winner and the Price Procedure. An additional statistical contrast run on difference scores ('after' minus 'before') also revealed that the procedural properties of the sophisticated procedures had a larger positive effect on the perceived fairness than the simple ones ($t=8.28$, $p<0.001$).

Given that we tested only seven fair-division procedures, one could wonder if the best solutions they generated were really the fairest and the most satisfactory. To assess this question, the solutions of the best division procedures were contrasted with the MaxSum solutions, that is, the solutions that maximized the sum of both participants ratings (see MaxSum in Figure 1). Results show that the MaxSum solutions were significantly more satisfactory than the Divide and Choose's solutions ($t(87)=3.23$, $p<0.002$) and significantly fairer than the Strict Alternation's solutions ($t(87)=2.61$, $p<0.02$).

Discussion

Following Rawls' theoretical notion of perfect procedural justice, this experiment tested the extent to which the distributive and procedural properties of the most sophisticated fair-division procedures (the Adjusted Knaster, the Adjusted Winner, the Compensation Procedure and the Price Procedure) were perceived as more satisfactory and fairer than simple (the Divide and Choose) and imperfect procedures (the Balanced Alternation, the Strict Alternation).

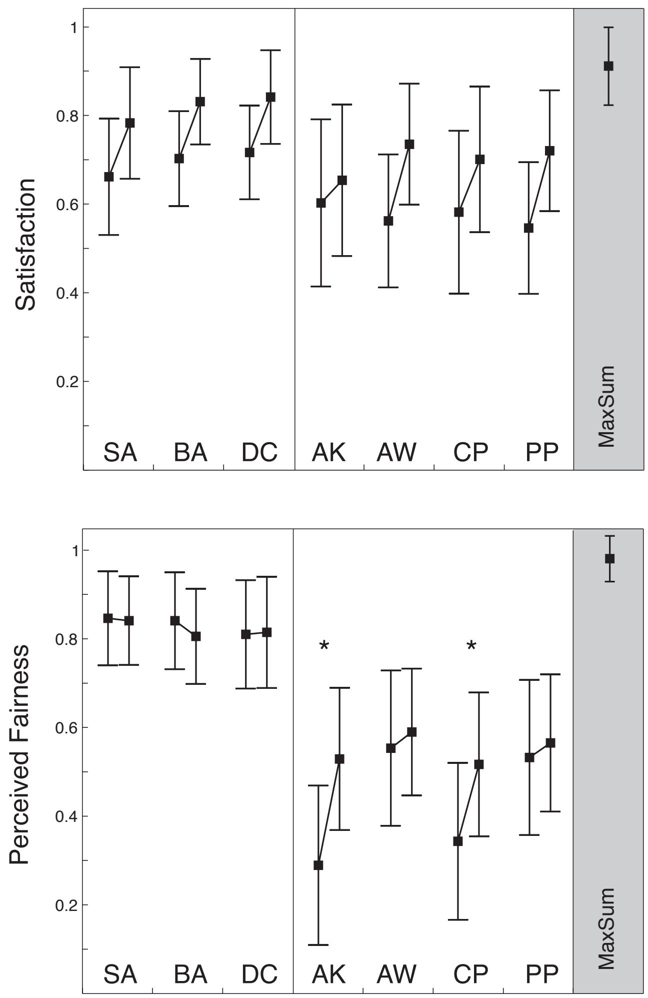

Figure 1. The average satisfaction and perceived fairness ratings (in percentiles) of seven fair-solutions generated by seven fair-division procedures, namely the Strict Alternation (SA), the Balanced Alternation (BA), the Adjusted Winner (AW), the Adjusted Knaster (AK), the Price Procedure (PP), the Divide and Choose (DC) and the Compensation Procedure (CP). Left-side error bars represent the first assessment of a division solution and the right-side error bars represent the assessment of a division following the implementation of a given division procedure. Error bars represent two standard errors. MaxSum represents the best tested alternative solution.

Thirty-nine pairs of students divided six $10 gift certificates between them using seven division procedures. They rated their satisfaction and their perceived fairness regarding the 64 possible discrete division solutions; then, they learned and executed each division procedure; and finally, they rated their satisfaction and their perceived fairness regarding the discrete division solution prescribed by each procedure. This experimental design allowed us to disentangle the effect of the procedural and distributive

[1] Note that there was no linear trend between the order in which the participants applied the procedures and the satisfaction (r=-0.08, *ns*) or the perceived fairness (r=-0.06, *ns*) ratings. Thus, we consider these repeated measures as being independent from each other.

properties of each fair-division procedure on the satisfaction and perceived fairness.

Contrarily to our hypothesis, the results show that perfect procedural justice does not really translate into the perception of a fairer and more satisfactory outcome and process. The results also refute the idea that recent improvements made to the procedural and distributive properties of fair-division procedures increased their positive impact on satisfaction and perceived fairness.

More specifically, the expected difference between sophisticated and imperfect (or simple) division procedures did not manifest itself in the satisfaction ratings. At the distributive level, we saw that the most satisfactory solutions came from simple and imperfect division procedures. At the procedural level, all division procedures had a similar positive impact on the satisfaction, suggesting that the mere application of a procedure, no matter which one, improved the participants' subjective well-being.

At first glance, the results on fairness suggest that the procedural properties of the most sophisticated division procedures did have a greater positive impact on the perception of fairness than simple and imperfect procedures. Notwithstanding, this positive procedural effect did not compensate for their weaknesses at the distributive level. By comparison, the simple and imperfect procedures produced solutions that were perceived as fairer even if it could not be ascribed to their procedural properties. Note that this absence of a procedural effect is unlikely to be due to a ceiling effect because there are more satisfactory and fairer solutions. In fact, of all the solutions rated by the dyads, 15.56% had a summed satisfaction higher than the Strict Alternation's solution and 16.18% had a summed perceived fairness greater than the Divide and Choose's solution—the two best division procedures.

Table 2. Multiple linear regressions between four distributive factors (column 1) and the two DVs. Regression coefficients, associated t-values (in parentheses) and statistical significance (asterisk) are shown.

Predictors	**Satisfaction**	**Perceived Fairness**
Object equality	0.27 (5.56)*	0.39 (9.25)*
Envy-free	-0.05 (-0.82)	0.03 (0.46)
Equitability	-0.07 (-1.26)	0.001 (0.03)
Efficiency	0.001 (0.05)	-0.28 (-4.18)*

These results suggest that the most sophisticated fair-division procedures have poor distributive justice, that is, they fail to select solutions that are perceived as fair and satisfactory. This is surprising given that they implement mathematically fairer solutions, which are also guaranteed against the most self-regarding and rational players. Hence, this raises the question whether the mathematical definitions of fairness match the implicit conception of fairness used by the participants. To answer this question, we performed a multiple linear regression between the criteria of justice met by the procedures' solutions (envy-freeness, equitability and efficiency) and each DV (satisfaction and perceived fairness).

Table 2 shows the regression coefficients of each factor and their associated t-values (in parentheses). The asterisks indicate significant t-values (p<0.0001). The results show that efficiency is negatively correlated with perceived fairness. This is consistent with the fact that inefficient procedures—the imperfect and simple ones—are also perceived as the fairest. This is also compatible with the observation of Herreiner and Puppe (2007) that a majority of participants are willing to sacrifice some efficiency in exchange of fairness. However, this result needs to be interpreted with caution because it probably does not generalize to all division contexts. In fact, although Fehr, Naef and Schmidt (2006) confirmed that most students were more concerned with equality than with efficiency, they also discovered that economics or business students were more concerned with efficiency than with equality.

Glancing at the best distributive solutions, we noticed that most exhibited an equal split of objects (i.e. three objects for each participant). In addition to the three standard criteria of justice, we thus added *object equality*—the distance to an equal split—in our analysis. We found that object equality is indeed positively correlated with both satisfaction and perceived fairness ratings. Object equality is usually not considered as a mathematical criterion of justice since it does not account for subjective preferences toward the objects. In fact, object equality does not entail envy-freeness, equitability or efficiency. Even if it is hard at this point to explain this preference for object equality over envy-freeness, equitability and efficiency, these results illustrate the mismatch between the implicit conception of fairness of humans and the mathematical conception of fairness.

Altogether, the results of this experiment show that the solutions from fair-division procedures could still be improved and that future work on fair-division should consider the psychological determinants of distributive and procedural fairness.

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