Motion-blur illusions

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Abstract. The still-radii illusion, the figure-of-eight illusion, the band-of-heightened-intensity illusion and the dark-blurred-concentric-circles illusion have remained, until now, isolated relatively illexplained phenomena. A single algorithmic model is proposed which explains these four visual illusions. In fact, this model predicts phenomena produced by motion of any gray-shaded patterns relative to the eyes (termed 'motion-blur illusions'). Results of a computer simulation of the model are presented. A novel instance of the proposed class of illusions, which can be readily experienced by the reader, is introduced to illustrate the generality of the model.

1 The still-radii illusion

Charles S W Cobbold (1881) was the first to give account of the still-radii illusion [named after Junge's (1963a) moving-radii illusion]. To elicit it one must move linearly in the frontoparallel plane a set of black concentric circles drawn on a white background (see figure 1a) relative to one's eyes. This is best achieved by eye-tracking the tip of a pencil moving at uniform speed across the pattern.

The still-radii illusion is characterised by a fan-like clear structure (perceived lying against a smeared background) orthogonal to the direction of motion.⁽¹⁾ Cobbold also provided an explanation of the still-radii illusion based on visual persistence:

"During vertical motion, the images of the more or less horizontal black and white bands are constantly replacing one another upon [the same regions of] the retina, each becoming confused with the impression immediately preceding it, and thus producing the blurred appearance noticed.... The reverse is the case ... in which the black and white bands are practically vertical, and coincide with the direction of motion; their images ... remain clearly defined and unaffected by the vertical movement.... The clearly-defined narrow sector [of the concentric circles] will then be seen extending horizontally... while the upper and lower portions of the disc appear somewhat ... blurred." (page 77)

⁽¹⁾ What we call the still-radii illusion must be distinguished from the revolving-propeller effect (eg Wade 1978). In fact, Wade made this duality quite explicit: "An *analogue* [italics added] to this [revolving-propeller effect]" he wrote "can be demonstrated by *moving the patterns relative to the eye* [italics added]: if the concentric circles are moved vertically up and down, two clear horizontal fans ... are seen, with the rest of the pattern slightly blurred." (page 33) The still-radii illusion, is, precisely, this 'analogue' illusion. For one thing, while the revolving-propeller effect is more highly correlated with the lens curvature fluctuation than with the eye movements (eg Millodot 1968), the still-radii illusion is perfectly correlated with eye movements (eg Cobbold 1881), and it is unaffected by lens paralysis (anonymous referee). The distinction is also mirrored in the distinct histories of the two illusions (although they may have a somewhat 'blurred' origin in Purkinje's 1823 work). The revolving-propeller effect was studied by Helmholtz (1856), Campbell and Robson (1958), Pritchard (1958), Evans and Marsden (1966), Millodot (1968), Wade (1978, 1982), etc. And the still-radii illusion was studied by Cobbold (1881), Bowditch and Hall (1882), Piéron (1901), and Junge (1963a, 1963b).



Figure 1. (a) Concentric circles; (b) ray figure; (c) grating.

Cobbold's account of the still-radii illusion implies two processing stages: first, the continuous projections of the pattern upon the retina is transformed into discrete 'impressions', and, then, two successive 'impressions' are 'confused' into an 'appearance'. To an observer, these 'appearances' should not be differentiable from the pattern represented in figure 1a when set in linear motion in the frontoparallel plane.

Cobbold added nothing about the first stage of processing implied by his explanation. It therefore remains very intuitive, no detail being given concerning the crucial 'confusing' process of the second stage of processing (affecting the successive 'impressions'). In fact, it only allows predicting that the more parallel a line in the pattern represented in figure 1a is to the movement of the eyes, the less blurred it will appear. In no way does it allow predicting the absolute degree of blurring of a line in the pattern, nor does it allow predicting the various extents of the blur for the various lines of the pattern.

2 The figure-of-eight illusion

Donald M MacKay (1958) was the first to report that when a ray figure (see figure 1b) is moved linearly in the frontoparallel plane relative to the eyes, blurred concentric 'figures of eight' perpendicular to the direction of the motion are perceived against a clear background. Again, the best way to produce the illusion is to eye-track the tip of

a pencil moving at uniform speed across the pattern. MacKay (1958, 1961) proposed an explanation of the figure-of-eight illusion which he generalised to phenomena (moiré effects, as he called them) produced by motion of all periodic black-and-white figures in the frontoparallel plane. Since the pattern represented in figure 1a is a periodic blackand-white pattern, MacKay's model should also account for the still-radii illusion.

Although MacKay never referred to Cobbold, his explanation can be understood as an elucidation of Cobbold's. Like Cobbold, MacKay proposed two stages of processing, which involve highly similar processes: first, the continuous projection of a black-andwhite periodic pattern upon the retina is transformed into discrete 'images', and, then, the 'images' are 'superposed', causing a result which should not be distinguishable, to an observer, from the moving stimulus. As opposed to Cobbold, however, who provided no further detail about the first stage of processing, MacKay stated clearly that the "[images] are displaced replicas of the [physical] pattern" (1958, page 362). The difference between the 'images' results from the movement of the pattern relative to the eyes. In other words, the 'images' are instantaneous snapshots of the visual world taken at different loci "as the eyes move" (MacKay 1961).

The second stage of processing in MacKay's theory is based on the combination of two 'successive images' or 'impressions' as is Cobbold's, but a much clearer description of how these two 'successive images' are combined is now provided: the two 'successive images' are 'superposed'. MacKay appeared to indicate that the "superposition of [two] successive images" is equivalent to the physical superposition of an "[image] with the transparency of itself" (1958, page 362). Thus, a white point (which is transparent on a transparency) superposed on a white point appears white, a white point superposed on a black point appears black, whilst a black point superposed on a white point appears black, and a black point superposed on a black point appears black.⁽²⁾ Whenever a periodic pattern (like the pattern represented in figure 1a or that represented in figure 1b, for instance) is superposed on a copy of itself in that fashion, moiré patterns are produced (eg Oster and Nishijima 1963). This is the reason why MacKay chose to call this class of phenomena 'moiré effects'.

The effects produced by the superposition of the patterns represented in figure 1a or figure 1b with transparencies of themselves are quite reminiscent of the still-radii illusion and the figure-of-eight illusion (eg Gregory 1990; Spillmann 1993), respectively. However, MacKay's model falls short of accounting for the 'blurred appearance' noticed by Cobbold in the still-radii illusion (moiré patterns are black or white).

3 The band-of-heightened-intensity and the dark-blurred-concentric-circles illusions

Babington Smith (1964) was the first to report that when a grating (see figure 1c) is rotated slowly relative to the eyes (between 0.175 and 1.047 rad s⁻¹) in the frontoparallel plane, a band of heightened intensity (lying against a smeared background) centred on the origin of the rotation is perceived at an angle with the lines of the grating. In fact, the so-called 'band of heightened intensity' is neither of 'decreased' intensity nor of 'heightened' intensity; it is simply more contrasted (Wade 1974). The speed of rotation is crucial to the production of the phenomenon. By rotating the pattern represented in figure 1c faster (beyond 10π rad s⁻¹) in the frontoparallel plane, Wade observed (1972) a completely different effect featuring 'dark blurred concentric circles' or, more specifically, alternating light and dark concentric circles that become less and less contrasted as their radii increase.

Since the pattern represented in figure 1c is a black-and-white periodic pattern, MacKay's model of moiré effects should account for the band-of-heightened-intensity and the dark-blurred-concentric-circles illusions (following Wade's 1972 description of the illusion).

⁽²⁾ This type of superposition mimics the behaviour of a logical 'and' with white corresponding to 'true' and black corresponding to 'false'.

The effect produced by the superposition of the pattern represented in figure 1c with a transparency of itself is, in fact, somewhat reminiscent of the band-of-heightenedintensity illusion. However, MacKay's theory cannot explain the dark-blurred-concentric-

intensity illusion. However, MacKay's theory cannot explain the dark-blurred-concentric-circles illusion. Covering the pattern represented in figure lc with a transparency of itself and rotating it through all the possible moiré patterns convincingly shows that none possesses the slightest resemblance to the dark-blurred-concentric-circles illusion. Smith (1964) proposed that the band-of-heightened-intensity illusion is the result of "the simultaneous perception of what is seen over a measurable period of time". Smith's proposition involves a single processing stage: the continuous projection of the rotating pattern upon the retina over a certain period of time is 'simultaneously perceived'. The result of this 'simultaneous perception' should not be differentiable, to an observer, from the pattern represented in figure 1c set in rotation. How this 'simultaneous perception' is achieved is so vague in Smith's explanation that it is difficult to see how anything could be predicted from it.

Barbur (1980) gave a much more satisfying description of this 'simultaneous perception'. According to him, the perception at a given receptor is the average of the sampled luminance at that receptor over a period p. Building upon this 'averaging hypothesis', Barbur elaborated a formal model accounting for part of Smith's band-ofheightened-intensity illusion.

Independently, Glünder (1987) showed that a time-integrating system (eg the human visual system) exposed to a suitably chosen frontoparallel rotating spatial frequency (a sinusoidally modulated luminance grating) approximates its 'temporal transfer function' (TTF) in the frequency domain. The TTF can be understood as a linear filter applied to a visual input signal. If the impulse response of the filter is chosen as 1/p over the period p, then the time-integrating system averages the input signal over p. Such a time-integrating system is thus equivalent to the one postulated by Barbur (1980). Now, the Fourier transform of that particular TTF (its representation in the frequency domain) is highly similar to the band-of-heightened-intensity illusion.

4 An algorithmic model of motion-blur illusions

We propose a general model of motion blur which has a bearing on phenomena produced by the motion of any gray-shaded patterns relative to the eyes, and not only on the band-of-heightened-intensity illusion as in Barbur's (1980), and Glünder's (1987) cases. We refer to these phenomena as 'motion-blur illusions'. Glünder (1987) distinguishes two types of time-integrating systems: those for which p is equal to the time of observation (eg a camera), and those for which p is smaller than the observa-tion time (eg a cine-camera, the human visual system). A time-integrating system of the second type can always be reduced to finite or infinite series of time-integrating systems of the first type (eg a cine-camera could be reduced to a finite series of cameras). Time-integrating systems of the first type can be thought of as building blocks of time-integrating systems of the second type. Our model will be concerned with the building blocks of the human visual time-integrating system and will be formulated at the 'representation and algorithm' level of understanding (Marr 1982).

The model will be presented in three steps. First (section 4.1), the problem of computing the projection of a gray-shaded image onto the retinal plane *at any single moment* (ie no integration over time yet) will be addressed. The problem of extending this algorithm to computing the integration *over time*, of the continuous projection of the stimulus pattern onto the retinal surface will then be addressed (section 4.2), completing the presentation of the model as such. The third step (section 4.3) will be concerned with the presentation of the model at work, as it is tested with various gray-shaded images of interest in the present context.

4.1 Projecting the visual world upon the retina

Since we wish to provide a general model of motion-blur illusions we must conceive a way of projecting the visible portion of the visual world upon the retina which can accommodate any type of motion. Our formal visual world (W) possesses four continuous dimensions: three spatial dimensions (X, Y, Z) (see 'visual world' in figure 2) and one temporal dimension (T).

We will restrict our analysis to a W where all points are invisible except for the gray-shaded points lying on one plane-the image (see 'Image' in figure 2). We have associated each gray shade with a positive real number corresponding to its relative position on a continuous gray scale (anywhere from black to white). Our gray scale is proportional to the luminance scale. The shade of any particular point on the image is given by the function

$$I(u,v)$$
,

(1)

where (u, v) is the location of that point relative to the image. All the pages of this article can be considered portions of such I functions.



Figure 2. Eye in the spatial visual world.

Thus, we have points located within a (U, V)-based 'private' or 'objective' coordinate system [ie irrespective of the visual world (X, Y, Z)-based coordinate system] and associated with a gray shade via a function I. When such a gray-shaded image is placed in the visual world W, its (U, V)-based point coordinates must be transformed into (X, Y, Z)-based point coordinates. Allowing for any type of motion in W, this can be achieved by using three standard motion functions. A simple linear motion in the frontoparallel plane as required to produce the still radii illusion, and the figure-ofeight illusion, for instance, could be expressed as follows:

$$x = u + k_X + k_S(t - k_T),$$
 (2)

$$y = v + k_Y, \tag{3}$$
$$z = k_Z, \tag{4}$$

where k_T corresponds to the value of t at the beginning of the computation; k_X , k_Y , and k_Z are the three coordinates of point (0,0) on the image in W at t equal to k_T ; and k_S corresponds to the constant speed of displacement along the X axis of W.

With the (u, v) coordinates of the points of the image *transformed*, at any moment, into (x, y, z) coordinates, we can now address the issue of computing the *projection* of that gray-shaded image onto the retinal plane. Our formal eye has a visual field of 90 deg, and, in order to respect the biconvex lens properties of the optics of the human eye, it is composed of two invisible bounded planes, embodied in W, and parallel to the plane defined by the X and Y axes of W (see 'Eye' in figure 2). The sides of the smaller bounded plane, called the 'Retina', are all of length n, with the intersection of its two diagonals being located at (0,0,0) in W. Thus, all retinal points (receptors) lie at z = 0, and take the form $(r_X, r_Y, 0)$, with $-\frac{1}{2}n \leq r_X$, $r_Y \leq \frac{1}{2}n$. The sides of the larger bounded plane, called the 'Pseudo lens', are all of length 2n; this larger plane being separated from the retina by a distance of $\frac{1}{2}n$, the intersection of its two diagonals is located at $(0, 0, \frac{1}{2}n)$ in W.

The point (x, y, z) on the image projects, at any moment, onto point $(r_x, r_y, 0)$ on the eye's retina following a *receptor line* passing through point $(2r_x, 2r_y, \frac{1}{2}n)$ on the pseudo lens (see 'Receptor line' in figure 2). Therefore, all we need in order to be able to compute the projection is the parametric equations of receptor lines, which are

$$x = r_X(2z/n+1),$$
 (5)

$$y = r_Y(2z/n+1),$$
 (6)

with $-\frac{1}{2}n \leq r_X$, $r_Y \leq \frac{1}{2}n$. Since we deal with half-lines and not with full lines, we must put a restriction on equations (5) and (6): only the portions of receptor lines on the positive side of the Z axis are to be considered.

All that remains to be done then, is to combine this *projection* process, characterised by the above 'receptor-line parametric equations' [equations (5) and (6)], and the *transformation* process described earlier as characterised by the 'standard motion functions' [and exemplified by equations (2), (3), and (4)]. This can be done by creating two functions, $F_U(r_X, r_Y, t)$ and $F_V(r_X, r_Y, t)$, which we will call the 'motion intersection functions', and which, in the case of the simple frontoparallel horizontal linear motion functions described in equations (2), (3), and (4), will yield [by combining equations (2), (3), (4), (5), and (6)] the following equations

$$u = r_X (2k_Z/n + 1) - k_X - k_S (t - k_T),$$
(7)

$$v = r_Y (2k_Z/n + 1) - k_Y.$$
(8)

Finally, by replacing u and v in equation (1) with their respective expressions in equations (7) and (8), we obtain the shade sampled by the receptor beginning at the point $(r_x, r_y, 0)$ in W at any moment.

It is important to note that *integration over time* has been, to this point, of no concern whatsoever. The *motion intersection functions* exclusively concern the geometry of how, at any single moment t, gray-shaded points given their motion relative to an eye, project onto this eye's retina.

Let us now turn precisely to this topic of the integration of gray shades over time—to the heart, in fact, of the present attempt to establish a computational rationale for motion-blur illusions.

4.2 Integrating the continuous projection of the visual world upon the retina

For integrating over time the gray-shaded image projected onto the retinal plane, we chose Barbur's (1980) 'averaging hypothesis'. In other words, a gray-shaded image set in motion in W should not be differentiable, to an observer, from the average of the

continuous projection of that image upon the retina over a fixed period p. Formally,

$$\frac{1}{p} \int_{k_T}^{k_T + p} I(F_U, F_V) \mathrm{d}t \tag{9}$$

for $-\frac{1}{2}n \leq r_X$, $r_Y \leq \frac{1}{2}n$.

4.3 Computer simulations of the model

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We performed a computer simulation of the proposed model using the motion intersection functions of the simple horizontal linear motion in the frontoparallel plane described above and the I functions or, rather, the portions of I functions represented in figures 1a and 1b. The program produced the patterns represented in figures 3a and 3b, respectively. These patterns are quite similar to the still-radii illusion and to the figure-of-eight illusion. The proposed model predicts that they should not be differentiable, to an observer, from the patterns represented in figures la and lb filling approximately 90 deg of his visual field set in a frontoparallel linear motion of about 3.6 deg p^{-1} relative to his eyes. If p was equal to 100 ms (eg Patterson 1990), that speed would be equal to 36 deg s⁻¹ approximately.⁽³⁾ From now on, we will use this conservative estimate of p.

Equations (10) and (11) are the motion intersection functions of a simple rotation in the frontoparallel plane about the central receptor of the retina

$$u = (2k_Z/n + 1)\{r_X \cos[k_S(t - k_T)] - r_Y \sin[k_S(t - k_T)]\} - k_X, \qquad (10)$$

$$v = (2k_Z/n + 1)\{r_X \sin[k_S(t - k_T)] + r_Y \cos[k_S(t - k_T)]\} - k_Y, \qquad (11)$$



Figure 3. The result of the application of the proposed model on the pattern represented in (a) figure 1a and (b) figure 1b with a horizontal linear motion in the frontoparallel plane.

⁽³⁾ The proposed model is not based on involuntary eye movements. If the human visual system always works as stated by the proposed model then any type of motion of the eye relative to gray-shaded images would generate some blurring. Now, under the heading 'involuntary eye movements', one usually includes: microsaccades, drifts, and tremors (Yarbus 1967). All of these eye movements are linear; hence they cannot account for the band-of-heightened-intensity and the dark-blurred-concentric-circles illusions. The retina undergoes linear displacement as large as 40 min visual angle during microsaccades. This is about 10 times more than during the average drift (over p, of course), and more than 60 times more than during tremors. Nonetheless, it is not enough for the proposed model to produce any noticeable 'radii' in the concentric circles represented in figure 1a filling 90 deg of the visual field.



Figure 4. The result of the application of the proposed model on the pattern represented in figure 1c with a rotation of (a) 0.06 rad p^{-1} and (b) 2π rad p^{-1} , respectively.

where k_s corresponds to the constant speed of the rotation. We performed another computer simulation of the proposed model using these motion intersection functions and the portion of *I* function represented in figure 1c, and the program generated the patterns represented in figures 4a and 4b. These patterns are very similar to the band-of-heightened-intensity illusion and to the dark-blurred-concentric-circles illusion, respectively.⁽⁴⁾ The proposed model predicts that the patterns represented in figures 4a and 4b should not be differentiable, to an observer, from the pattern represented in figure 1c filling approximately 90 deg of his visual field, and set in a frontoparallel rotation of about 0.6 rad s⁻¹ and 20 π rad s⁻¹, respectively.

Equations (12) and (13) are the motion intersection functions of a simple back or forth motion perpendicular to the frontoparallel plane

$$u = r_X \{ 2[k_S(t - k_T) + k_Z] / n + 1 \} - k_X$$
(12)



Figure 5. The result of the application of the proposed model on the pattern represented in figure 1a with a back or forth motion perpendicular to the frontoparallel plane.

⁽⁴⁾ The program used to simulate the algorithmic model trims the boundaries of the predicted patterns. This trimming—especially noticeable in the pattern represented in figure 4b—is not a prediction of the model.

and

$$v = r_Y \{ 2[k_S(t - k_T) + k_Z]/n + 1 \} - k_Y,$$
(13)

where k_s corresponds to the constant speed of the linear motion. We performed a final computer simulation of the proposed model using these motion intersection functions and the portion of *I* functions represented in figure 1a, and we obtained the pattern represented in figure 5. The shape of the blurred portion of the image is quite reminiscent of that of a ghost. As far as we know, this is a new prediction. One can easily test it by moving back (frontoperpendicularly) relative to one's eyes at a speed of about 0.4 m s⁻¹ a corner of the pattern represented in figure 1a encompassing 90 deg of one's visual field at 0.5 m.

Acknowledgements. We wish to thank N J Wade and an anonymous referee for their insightful reviews. We are indebted also to this anonymous referee for having performed a critical experiment showing that the still-radii illusion is unaffected by lens paralysis. The first author was supported by a scholarship from the Natural Sciences and Engineering Research Council of Canada (NSERC) during this research.

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