Motion Blur Illusions

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Abstract

The Still Radii Illusion (Cobbold 1881), the Figure of Eight Illusion (MacKay 1958), the Band of Heightened Intensity Illusion (Smith 1964) and the Dark Blurred Concentric Circles Illusion (Wade 1972) have remained, until now, isolated relatively ill-explained phenomena. A single algorithmic model is proposed which explains these four visual illusions. In fact, this model predicts phenomena produced by motion of any gray shaded patterns relative to the eyes (termed "Motion Blur Illusions"). Results of a computer simulation of the model are presented. A novel instance of the proposed class of illusions, which can be readily experienced by the reader is introduced to illustrate the generality of the model.

Motion Blur Illusions

1. The Still Radii Illusion

Charles S. W. Cobbold (1881) was the first to give account of the Still Radii Illusion (named after Junge's 1963a Moving Radii Illusion.). To elicit it one must move linearly in the fronto-parallel plane a set of black concentric circles drawn on a white background (see figure 1a) relative to one's eyes. This is best achieved by eye tracking the tip of a pencil moving at uniform speed across the pattern.

Insert figure 1 about here

The Still Radii Illusion is characterized by a fan-like clear structure (perceived lying against a smeared background) orthogonal to the direction of motion[1]. Cobbold also provided an explanation of the Still Radii Illusion based on visual persistence:

> During vertical motion, the images of the more or less horizontal black and white bands are constantly replacing one another upon [the same regions of] the retina, each becoming confused with the impression immediately preceding it, and thus producing the blurred appearance noticed.... The reverse is the case ... in which the black and white bands are practically vertical, and coincide with the direction of motion; their images ... remain clearly defined and unaffected by the vertical movement.... The clearly-defined narrow sector [of the concentric circles] will then

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[1]What we call the Still Radii Illusion must be distinguished from the Revolving Propeller Effect (e.g., Wade 1978). In fact, Wade (1978) made this duality quite explicit: "An *analogue* [italics added] to this [Revolving Propeller Effect]" he wrote "can be demonstrated by *moving the patterns relative to the eye* [italics added]: if the concentric circles are moved vertically up and down, two clear horizontal fans ... are seen, with the rest of the pattern slightly blurred." (p. 33) The Still Radii Illusion is, precisely, this "analogue" illusion. For one thing, while the Revolving Propeller Effect seems to be more highly correlated with the lens curvature fluctuation than with the eye movements (e.g., Campbell & Robson 1958), the Still Radii Illusion is perfectly correlated with eye movements (e.g., Cobbold 1881), and it is unaffected by lens paralysis (anonymous referee). The distinction is, also, mirrored in the distinct histories of the two illusions (although they may have a somewhat "blurred" origin in Purkinje's 1832 work). The Revolving Propeller Effect was studied by Helmholtz (1856), Campbell & Robson (1958), Pritchard (1958), Evans & Marsden (1966), Millodot (1968), Wade (1978 and 1982), etc. And, the Still Radii Illusion was studied by Cobbold (1881), Bowditch & Hall (1882), Piéron (1901), and Junge (1963a and 1963b).

> be seen extending horizontally ... while the upper and lower portions of the disc appear somewhat ... blurred. (p. 77)

Cobbold's account of the Still Radii Illusion implies two processing stages: first, the continuous projection of the pattern upon the retina is transformed into discrete "impressions", and, then, two successive "impressions" are "confused" into an "appearance ". To an observer, these "appearances" should not be differentiable from the pattern represented in figure 1a when set in linear motion in the fronto-parallel plane.

Cobbold added nothing about the first stage of processing implied by his explanation. It therefore remains very intuitive, no detail being given concerning the crucial "confusing" process of the second stage of processing (affecting the successive "impressions"). In fact, it only allows predicting that the more parallel a line in the pattern represented in figure 1a is to the movement of the eyes, the less blurred it will appear. In no way does it allow predicting the absolute degree of blurring of a line in the pattern, nor does it allow predicting the various extents of the blur for the various lines of the pattern.

2. Figure of Eight Illusion

Donald M. MacKay (1958) was the first to report that when a ray figure (see figure 1b) is moved linearly in the fronto-parallel plane relative to the eyes, blurred concentric "figures of eight" perpendicular to the motion's direction are perceived against a clear background. Again, the best way to produce the illusion is to eye track the tip of a pencil moving at uniform speed across the pattern. MacKay (1958, 1961) proposed an explanation of the Figure of Eight Illusion which he generalized to phenomena (Moiré Effects, as he called them) produced by motion of all periodical black and white figures in the fronto-parallel plane. Since the pattern represented in figure 1a is a periodical black and white pattern, MacKay's model should also account for the Still Radii Illusion.

Although MacKay never refered to Cobbold, his explanation can be understood as an elucidation of Cobbold's. Like Cobbold, MacKay proposed two stages of processing, which involve highly similar processes: first, the continuous projection of a black and white periodical pattern upon the retina is transformed into discrete "images", and, then, the "images" are "superposed", causing a result which should not be distinguishable, to an observer, from the moving stimulus. As opposed to Cobbold, however, who provided no further detail about the first stage of processing, MacKay stated clearly that the " [images] are displaced replicas of the [physical] pattern" (1958, p. 362). The difference between the "images" results from the movement of the pattern relative to the eyes. In other words, the "images" are instantaneous snapshots of the visual world taken at different loci "as the eyes move" (MacKay 1961).

The second stage of processing in MacKay's theory is based on the combination of two "successive images" or "impressions" as is Cobbold's, but a much clearer description of how these two "successive images" are combined is now provided: the two "successive images" are "superposed". MacKay appeared to indicate that the "superposition of [two] successive images" is equivalent to the physical superposition of an "[image] with the transparency of itself" (1958, p. 362). Thus, a white point (which is transparent on a transparency) superposed on a white point appears white, a white point superposed on a black point appears black, whilst a black point superposed on a white point appears black, and a black point superposed on a black point appears black[2]. Whenever a periodical pattern (like the pattern represented in figure 1a or that represented in figure 1b, for instance) is superposed to a copy of itself in that fashion, moiré patterns are produced (e.g., Oster & Nishijima 1963). This is the reason why MacKay chose to call this class of phenomena "Moiré Effects".

The effects produced by the superposition of the patterns represented in figure 1a or figure 1b with transparencies of themselves are quite reminiscent of the Still Radii Illusion and the Figure of Eight Illusion (e.g., Spillmann 1993; Gregory 1990), respectively. However, MacKay's model falls short of accounting for the "blurred appearance" noticed by Cobbold in the Still Radii Illusion (Moiré patterns are black or white.).

3. The Band of Heightened Intensity and the Dark Blurred Concentric Circles Illusions

Babington Smith (1964) was the first to report that when a grating (see figure 1c) is rotated slowly relative to the eyes (between 0.175 and 1.047 rad s^{-1}) in the fronto-parallel plane, a "band of heightened intensity" (lying against a smeared background) centered on the origin of the rotation is perceived at an angle with the grating's lines. In fact, the so-called "band of heightened intensity" is neither of "decreased" intensity nor of "heightened" intensity; it is simply simply more contrasted (Wade 1974). The speed of rotation is crucial to the production of the phenomenon. By rotating the pattern represented in figure 1c faster (beyond 10[illegible] rad s^{-1}) in the fronto-parallel plane, Wade observed (1972) a completely different effect featuring "dark blurred concentric circles" or, more specifically,

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[2]This type of superposition mimics the behavior of a logical "and" with white corresponding to "true" and black corresponding to "false".

alternatively light and dark concentric circles that become less and less contrasted as their radii increase.

Since the pattern represented in figure 1c is a black and white periodical pattern, MacKay's model of Moiré Effects should account for the Band of Heightened Intensity and the Dark Blurred Concentric Circles Illusions (following Wade's 1972 description of the illusion).

The effect produced by the superposition of the pattern represented in figure 1c with a transparency of itself is, in fact, somewhat reminiscent of the Band of Heightened Intensity Illusion. However, MacKay's theory cannot explain the Dark Blurred Concentric Circles Illusion. Covering the pattern represented in figure 1c with a transparency of itself and rotating it through all the possible moiré patterns convincingly shows that none possess the slightest resemblance with the Dark Blurred Concentric Circles Illusion.

Smith (1964) proposed that the Band of Heightened Intensity Illusion is the result of "the simultaneous perception of what is seen over a measurable period of time" (p. 27A). Smith's proposition involves a single processing stage: the continuous projection of the rotating pattern upon the retina over a certain period of time is "simultaneously perceived". The result of this "simultaneous perception" should not be differentiable, to an observer, from the pattern represented in figure 1c set in rotation. How this "simultaneous perception" is achieved is so vague in Smith's explanation that it is difficult to see how anything could be predicted from it.

Barbur (1980) gave a much more satisfying description of this "simultaneous perception". According to him, the perception at a given receptor is the average of the sampled luminance at that receptor over a period p. Building upon this "averaging hypothesis", Barbur elaborated a formal model accounting for part of Smith's Band of Heightened Intensity Illusion.

Independently, Glünder (1987) showed that a time-integrating system (e.g., the human visual system) exposed to a suitably chosen fronto-parallel rotating spatial frequency (i.e., a sinusoidally modulated luminance grating) approximates its "temporal transfer function" (TTF) in the frequency domain. The TTF can be understood as a linear filter applied to a visual input signal. If the impulse response of this filter is chosen as $1/p$ over the period p, then the time-integrating system averages the input signal over p. Such a time-integrating system is, thus, equivalent to the one postulated by Barbur (1980). Now, the Fourier transform of that particular TTF (its representation in the frequency domain) is highly similar to the Band of Heightened Intensity Illusion.

4. An Algorithmic Model of Motion Blur Illusions

We propose a general model of motion blur which has a bearing on all the phenomena produced by the motion of gray shaded patterns relative to the eyes, and not only on the Band of Heightened Intensity Illusion as in Barbur's (1980) and Glünder's (1987) cases. We refer to these phenomena as "Motion Blur Illusions". Glünder (1987) distinguishes two types of time-integrating systems: those for which p is equal to the time of observation (e.g., a camera), and those for which p is smaller than the observation time (e.g., a cine-camera, the human visual system). A time-integrating system of the second type can always be reduced to finite or infinite series of time-integrating systems of the first type (e.g., a cine-camera could be reduced to a finite series of cameras). Time-integrating systems of the first type can be thought of as building blocks of time-integrating systems of the second type. Our model will be concerned with the building blocks of the human visual time-integrating system and will be formulated at the "representation and algorithm" level of understanding (Marr 1982).

The model will be presented in three steps. First (section 4.1), the problem of computing the projection of a gray shaded image onto the retinal plane *at any single moment* (i.e. no integration over time yet) will be addressed. The problem of extending this algorithm to computing the integration *over time*, of the continuous projection of the stimulus pattern onto the retinal surface will then be addressed (section 4.2), completing the presentation of the model as such. The third step (section 4.3) will be concerned with the presentation of the model at work, as it is tested with various gray shaded images of interest in the present context.

4.1 Projecting the Visual World Upon the Retina

Since we wish to provide a general model of Motion Blur Illusions we must conceive a way of projecting the visible portion of the visual world upon the retina which can accommodate any type of motion. Our formal visual world (W) possesses four continuous dimensions: three spacial dimensions (X, Y, Z) (see "Visual World" in figure 2) and one temporal dimension (T).

Insert figure 2 about here

We will restrict our analysis to a W where all points are invisible except for the gray shaded points lying on one plane, the image (see "Image" in figure 2). We have associated each gray shade with a positive real number corresponding to its relative position on a continuous gray scale (anywhere from black to white). Our gray scale is proportional to the luminance scale. The shade of any particular point on the image is given by the function

$$I(u, v) \qquad (1)$$

where (u, v) is the location of that point relative to the image. All the pages of this article can be considered portions of such I functions.

Thus, we have points located within a (U, V)-based "private" or "objective" coordinate system (i.e. irrespective of the visual world (X, Y, Z)-based coordinate system) and associated with a gray shade via a function I. When such a gray shaded image is placed in the visual world W, its (U, V)-based point coordinates must be *transformed* into (X, Y, Z)-based point coordinates. Allowing for any type of motion in W, this can be achieved by using three standard motion functions. A simple linear motion in the fronto-parallel plane as required to produce the Still Radii Illusion and the Figure of Eight Illusion, for instance, could be expressed as follows:

$$x = u + k_X + k_S(t - k_T) \qquad (2)$$

$$y = v + k_Y \qquad (3)$$

$$z = k_Z \qquad (4)$$

where k_T corresponds to the value of t at the beginning of the computation; k_X, k_Y and k_Z are the three coordinates of point (0,0) on the image in W at t equal to k_T; and k_S corresponds to the constant speed of displacement along the X axis of W.

With the (u, v) coordinates of the points of the image *transformed*, at any moment, into (x, y, z) coordinates, we can now address the issue of computing the *projection* of that gray shaded image onto the retinal plane. Our formal eye has a visual field of 90 deg, and, in order to respect the biconvex lens properties of the optics of the human eye, it is composed of two invisible bounded planes, embodied in W, and parallel to the plane defined by the X and Y axes of W (see "Eye" in figure 2). The sides of the smaller bounded plane, called "the retina", are all of length n, with the intersection of its two diagonals being located at $(0, 0, 0)$ in W. Thus, all retinal points (receptors) lie at $z = 0$, and take the form $(r_X, r_Y, 0)$, with $-\frac{n}{2} \le r_X, r_Y \le \frac{n}{2}$. The sides of the larger bounded plane, called the "pseudo lens", are all of length $2n$; this larger plane being separated from the retina by a distance of $\frac{n}{2}$, the intersection of its two diagonals is located at $(0, 0, \frac{n}{2})$ in W.

The point (x, y, z) on the image projects, at any moment, onto point $(r_X, r_Y, 0)$ on the eye's retina following a *receptor line* passing through point $(2r_X, 2r_Y, \frac{n}{2})$ on the pseudo lens (see "Receptor Line" in figure 2). Therefore, all we need in order to be able to compute the projection is the parametric equations of receptor lines, which are

$$x = r_X\left(\frac{2}{n} z + 1\right) \qquad (5)$$

$$y = r_Y\left(\tfrac{2}{n} z + 1\right) \qquad (6)$$

with $-\frac{n}{2} \le r_X, r_Y \le \frac{n}{2}$. Since we deal with half-lines and not with full lines, we must put a restriction on equations 2 and 3: only the portions of receptor lines on the positive side of the Z axis are to be considered.

All that remains to be done then, is to combine this *projection* process, characterized by the above "receptor line parametric equations" (equations 5 and 6), and the *transformation* process described earlier as characterized by the "standard motion functions" (and examplified by equations 2, 3 and 4). This can be done by creating two functions, $F_U(r_X, r_Y, t)$, and $F_V(r_X, r_Y, t)$, which we will call the "motion intersection functions", and which, in the case of the simple fronto-parallel horizontal linear motion functions described in equations 2, 3 and 4, will yield (by combining equations 2,3,4,5 and 6) the following equations

$$u = r_X\left(\tfrac{2}{n} k_Z + 1\right) - k_X - k_S(t - k_T) \qquad (7)$$

$$v = r_Y\left(\tfrac{2}{n} k_Z + 1\right) - k_Y. \qquad (8)$$

Finally, by replacing u and v in equation 1 with their respective expressions in equations 7 and 8, we obtain the shade sampled by the receptor beginning at the point $(r_X, r_Y, 0)$ in W at any moment.

It is important to note that *integration over time* has been, to this point, of no concern whatsoever. The *motion intersection functions* exclusively concern the geometry of how, at any single moment t, gray shaded points given their motion relative to an eye, project onto this eye's retina.

Let us now turn precisely to this topic of the integration of gray shades over time, to the heart, in fact, of the present attempt to establish a computational rationale for motion blur illusions.

4.2 Integrating the Continuous Projection of the Visual World Upon the Retina

For integrating over time the gray shaded image projected onto the retinal plane, we chose Barbur's "averaging hypothesis" (1980). In other words, a gray shaded image set in motion in W should not be differentiable, to an observer, from the average of the continuous projection of that image upon the retina over a fixed period p. Formally,

$$\frac{1}{p}\int_{k_T}^{k_T+p} I(F_U, F_V).\,dt \qquad (9)$$

for $-\frac{n}{2} \le r_X, r_Y \le \frac{n}{2}$.

4.3 Computer simulations of the model

We performed a computer simulation of the proposed model using the motion intersection functions of the simple horizontal linear motion in the fronto-parallel plane described above and the I functions or rather, the portions of I functions represented in figures 1a and 1b. The program produced the patterns represented in figures 3a and 3b, respectively. These patterns are quite similar to the Still Radii Illusion and to the Figure of Eight Illusion. The proposed model predicts that they should not be differentiable, to an observer, from the patterns represented in figures 1a and 1b filling approximatively 90 deg of his visual field set in a fronto-parallel linear motion of about 3.6 deg p^{-1} relative to his eyes. If p was equal to 100 ms (e.g., Patterson 1990), that speed would be equal to 36 deg s^{-1} approximatively[3]. From now on, we will use this conservative estimate of p.

Insert figure 3 about here

Equations 10 and 11 are the motion intersection functions of a simple rotation in the fronto-parallel plane about the central receptor of the retina

$$u = \left(\tfrac{2}{n} k_Z + 1\right)\left\{r_X \cos\left[k_S\left(t - k_T\right)\right] \square r_Y \sin\left[k_S\left(t - k_T\right)\right]\right\} \square k_X \qquad (10)$$

$$v = \left(\tfrac{2}{n} k_Z + 1\right)\left\{r_X \sin\left[k_S\left(t - k_T\right)\right] + r_Y \cos\left[k_S\left(t - k_T\right)\right]\right\} \square k_Y \qquad (11)$$

where k_S corresponds to the constant speed of the rotation. We performed another computer simulation of the proposed model using these motion intersection functions and the portion of I function represented in figure 1c, and the program generated the patterns represented in figures 4a and 4b. These patterns are very similar to the Band of Heightened Intensity Illusion and to the Dark Blurred Concentric Circles Illusion, respectively[4]. The proposed model predicts that the patterns represented in figures 4a and 4b should not be

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[3]The proposed model *is not* based on involuntary eye movements. If the human visual system always works as stated by the proposed model then any type of motion of the eye relative to gray shaded images would generate *some blurring*. Now, under the heading "involuntary eye movements", one, usually, includes: microsaccades, drifts, and tremors (Yarbus 1967). All of these eye movements are linear, hence, they cannot account for the Band of Heightened Intensity and the Dark Blurred Concentric Circles Illusions. The retina undergoes linear displacement as large as 40 minutes of angle during microsaccades. This is about 10 times more than during the average drift (over p, of course), and more than 60 times more than during tremors. Nonetheless, it is not enough for the proposed model to produce any noticeable "radii" in the concentric circles represented in figure 1a filling 90 deg of the visual field.

[4]The program used to simulate the algorithmic model trims the boundaries of the predicted patterns. This trimming -- especially noticeable in the pattern represented in figure 4b -- is not a prediction of the model.

differentiable, to an observer, from the pattern represented in figure 1c filling approximatively 90 deg of his visual field, and set in a fronto-parallel rotation of about 0.6 rad s^{-1} and 20π rad s^{-1}, respectively.

Insert figure 4 about here

Equations 12 and 13 are the motion intersection functions of a simple back or forth motion perpendicular to the fronto-parallel plane

$$u = r_X \left\{ \frac{2}{n} \left[k_S \left(t - k_T \right) + k_Z \right] + 1 \right\} \square \, k_X \qquad (12)$$

$$v = r_Y \left\{ \frac{2}{n} \left[k_S \left(t - k_T \right) + k_Z \right] + 1 \right\} \square \, k_Y \qquad (13)$$

where k_S corresponds to the constant speed of the linear motion. We performed a final computer simulation of the proposed model using these motion intersection functions and the portion of I functions represented in figure 1a, and we obtained the pattern represented in figure 5. The shape of the blurred portion of the image is quite reminiscent of that of a ghost. As far as we know, this is a new prediction. One can easily test it by moving back (fronto-perpendicularly) relative to one's eyes at a speed of about 0.4 m s^{-1} a corner of the pattern represented in figure 1a encompassing 90 deg of one's visual field at 0.5 m.

Insert figure 5 about here

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Figure Captions

Figure 1a. Concentric circles.

Figure 1b. Ray figure.

Figure 1c. Grating.

Figure 2. Eye in the spatial visual world.

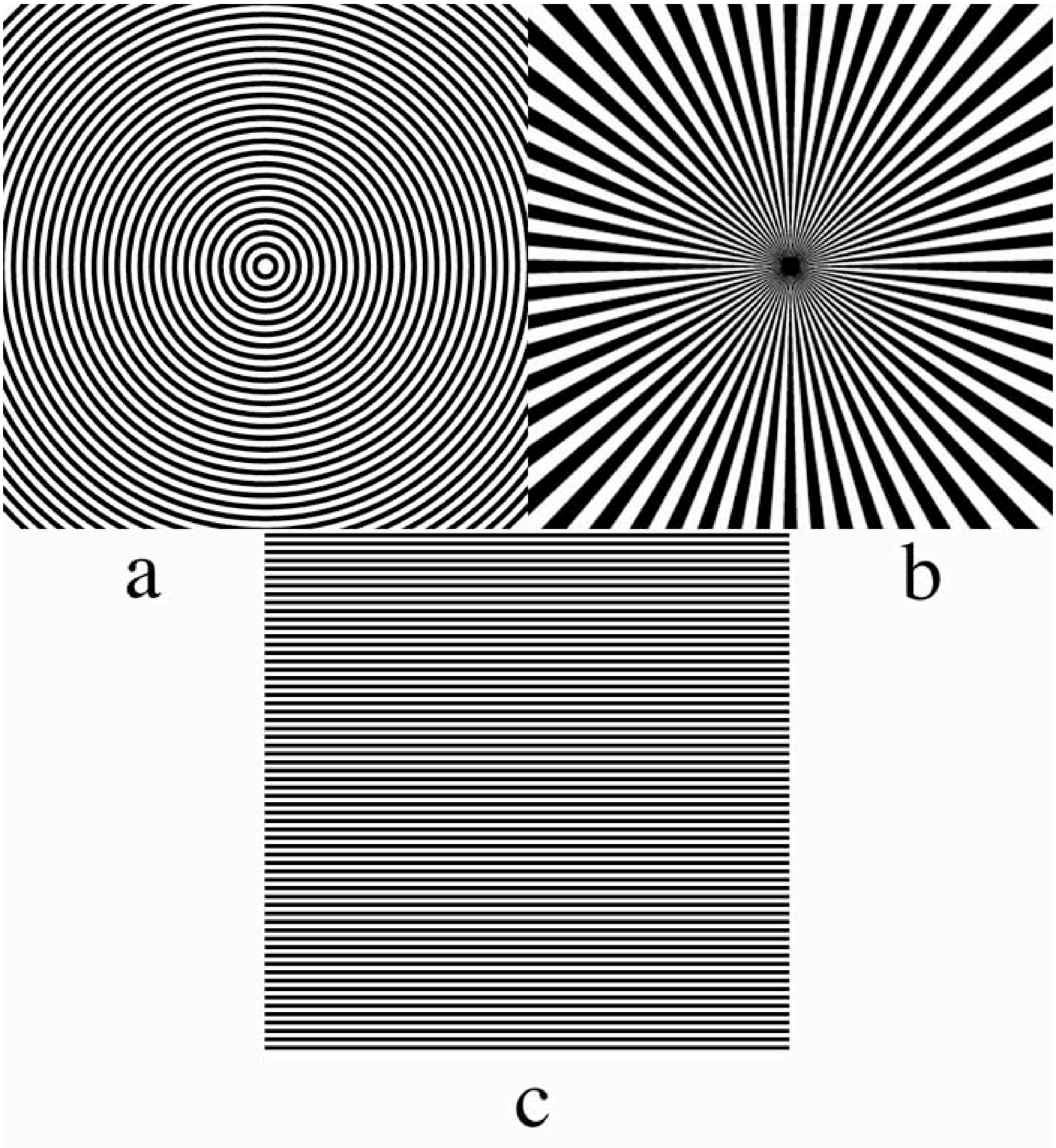
Figure 3a. The result of the application of the proposed model on the pattern represented in figure 1a with a horizontal linear motion in the fronto-parallel plane.

Figure 3b. The result of the application of the proposed model on the pattern represented in figure 1b with a horizontal linear motion in the fronto-parallel plane.

Figure 4a. The result of the application of the proposed model on the pattern represented in figure 1c with a rotation of 0.06 rad p^{-1} in the fronto-parallel plane.

Figure 4b. The result of the application of the proposed model on the pattern represented in figure 1c with a rotation of 2π rad p^{-1} in the fronto-parallel plane.

Figure 5. The result of the application of the proposed model on the pattern represented in figure 1a with a back or forth motion perpendicular to the fronto-parallel plane.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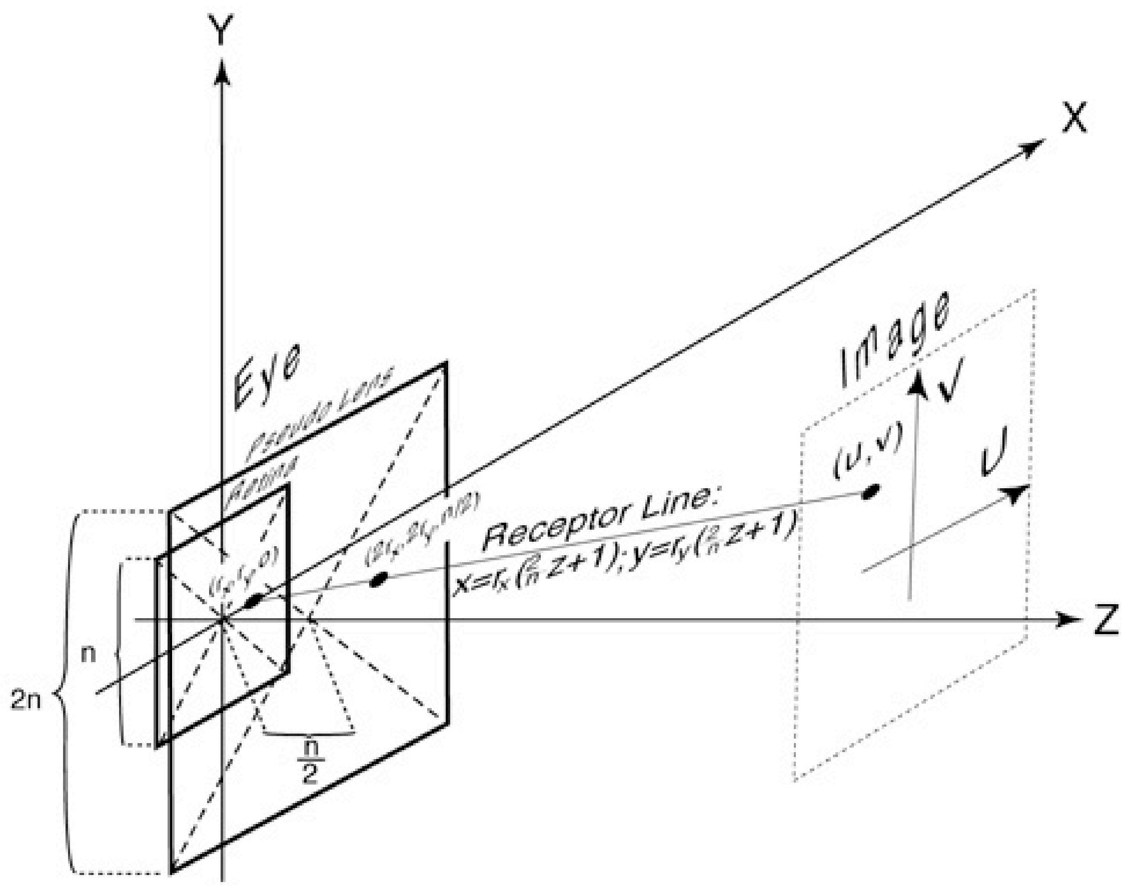
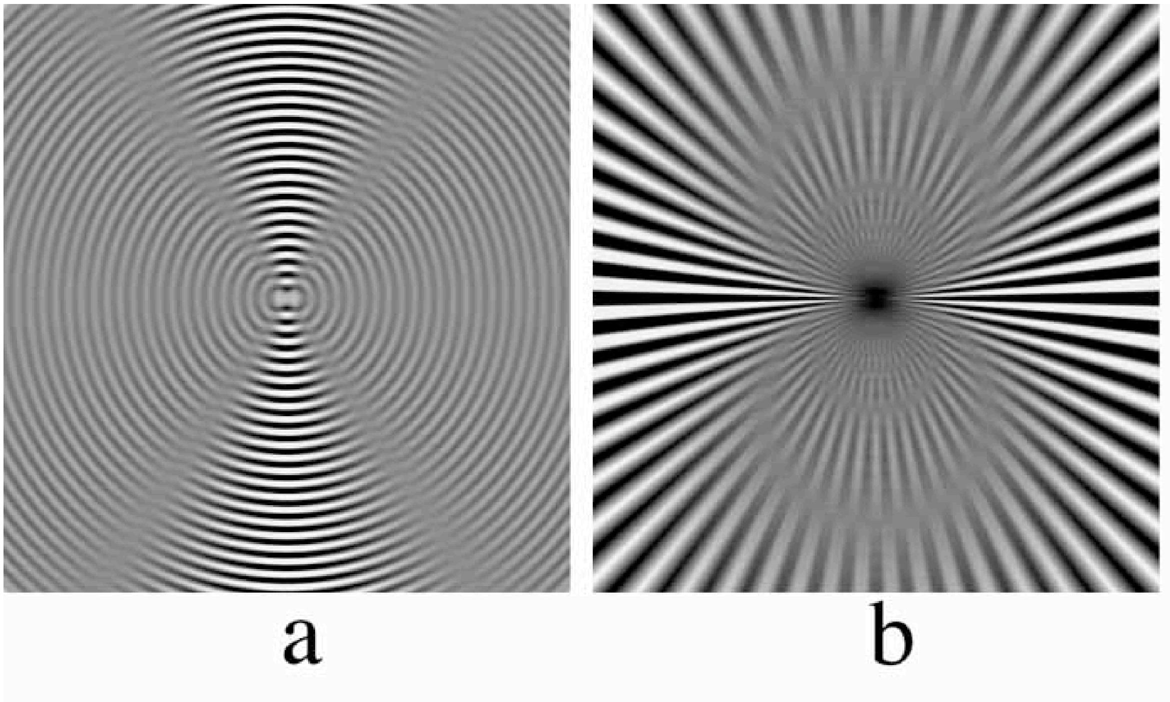
Spatial Visual World

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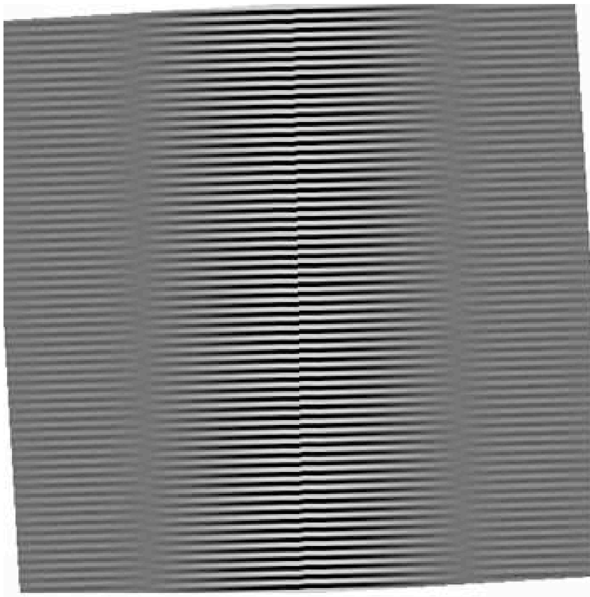

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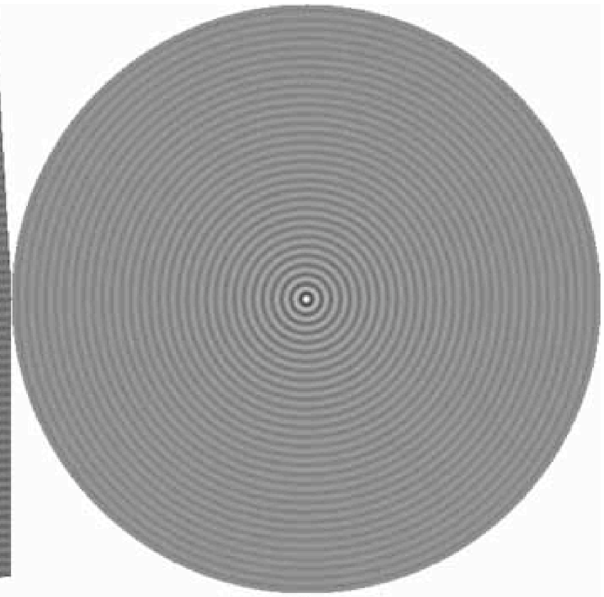

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