An Empirical Analysis of Nonstationarity in a Panel of Interest Rates with Factors[1]

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Abstract

This paper studies nonstationarities in a panel of Canadian and U.S. interest rates of different maturities and risk. We focus on methods which model the cross-sectional dependence within the panel as a linear dynamic factor model, and decompose our data into common and idiosyncratic components that we analyze in turn.

Our results suggest the presence of a single nonstationary factor in our panel. Since some of the idiosyncratic components are stationary, we conclude that these series are cointegrated. Finally, the dominant factors can be interpreted as level and slope factors as in the term structure literature.

1 Introduction

In earlier empirical studies, using standard methods, nominal interest rates of different maturities are typically found to be nonstationary and cointegrated (see for example Campbell and Shiller, 1987 and Evans and Lewis, 1994). On the other hand, early results in the nonstationary panel literature that suppose that the series in the panel are independent are more favorable to stationarity (see for example, Wu and Chen, 2001).

In this paper, we analyze a panel of 25 monthly Canadian and US interest rates of different maturities and risk. First, we document significant cross-sectional correlation among the series in the panel. To model the correlation, we employ a dynamic factor model. Then, our analysis focuses on a decomposition of the panel into common and idiosyncratic components in order to answer the following three questions: (i) is the observed data stationary or not and if it is nonstationary, is it because of nonstationary common components or nonstationary idiosyncratic components?; (ii) how many common factors are necessary to capture the cross-sectional correlations and how many of these common factors are nonstationary?; and (iii) do the estimated factors represent some observable variables of interest?

The first question will be answered by carrying out a series of panel unit root tests and stationarity tests that have been developed recently. Depending on the specification of the test, this will test whether the idiosyncratic or common components are nonstationary or not. The first part of the second question will be answered by using the panel information criteria proposed by Bai and Ng (2002), while the second part of the question will be answered using a combination of tests and a new information criterion developed by Bai (2004). Finally, the third question will be answered using the results in Bai (2004).

We find that interest rates are characterized by a single nonstationary factor and some stationary idiosyncratic components, in other words they are cointegrated as in Campbell and Shiller (1987) and Evans and Lewis (1994). Secondly, we find that the first two factors in our interest rate panel have interpretations as level and slope factors as in the term structure literature.

The paper is organized as follows. Section 2 introduces our data set and documents strong cross-sectional correlation. Sections 3 provides an overview of the methodology for the analysis of factors and our related empirical results. In Section 4, we discuss our analysis of the idiosyncratic components by focusing specifically on the application of panel unit root tests with factors. Finally, Section 5 concludes.

2 Data

This section introduces our panel of interest rates briefly. Throughout, we suppose that we have panel data z_{it} of individual i that is observed at time t. Let n and T denote the size of the cross section and time series dimensions, respectively. We model our panel using the decomposition among deterministic, common and idiosyncratic components as in Bai and Ng (2004):

$$z_{it} = d_{it} + c_{it} + u_{it}, \tag{1}$$

where d_{it} is the deterministic component, c_{it} the common component, and u_{it} the idiosyncratic component. In view of the type of data that we consider, the deterministic component will be restricted to an individual-specific intercept, $d_{it} = \alpha_i$. We model the common component, c_{it}, with a linear factor structure. Our goal is to characterize the stationary properties of the common and idiosyncratic components.

Except for the case where the common component is nonstationary while the idiosyncratic one is stationary (hence z_{it} is cointegrated), it is sometimes more convenient for characterizing the stationarity properties of z_{it} to express model (1) in an autoregressive form as in Moon and Perron (2004) :

$$\begin{aligned} z_{it} &= \alpha_i + z_{it}^0 && (2) \\ z_{it}^0 &= \rho_i z_{it-1}^0 + y_{it}, \end{aligned}$$

where y_{it} are unobservable error terms with a factor structure:

$$y_{it} = \beta_i' f_t + e_{it}, \tag{3}$$

where f_t are $K \times 1$ vectors of unobservable random factors, β_i are nonrandom factor loading coefficients, e_{it} are idiosyncratic shocks, and the number of factors K is unknown.

Because Bai and Ng (2004) work with the decomposition (1) directly, this allows them to consider testing separately the stationarity of the factors and the stationarity of the idiosyncractic component. This has two advantages: first, the factors are considered as objects of interest than can be analyzed, and second, cointegration among the panel units is allowed. We will follow this strategy rather closely.

Our panel consists of 14 monthly Canadian interest rates and 11 U.S. rates. These vary by both maturity and risk. The included Canadian rates are 1-, 3-, and 6-month T-bills, federal government bonds with a maturity of 1, 2, 3, 4, 7, and 10 years, commercial paper with a maturity of 1 month and 3 months and Scotia indices of yields on corporate bonds with mid-term and long-term maturities. The U.S. rates are Treasury securities with 3 months, 6 months, and 1, 2, 3, 5, 7, and 10 years to maturity, 1-month commercial paper, and Moody's indices of yields on corporate bonds with AAA and BAA ratings. The panel spans the January 1985-April 2004 period for a total of 232 observations for each rate.

Table 1 presents estimates of the short-run correlation matrix for this data.[1] We have divided the data into Canadian and U.S. rates. There are high correlations among yields with similar maturity in a given country. In particular, the correlation among long rates is very high as should be the case if the expectations hypothesis were true. There is much lower correlation within a country between short rates and long rates and across countries for the same maturity. For example, the 3-month Canadian T-bill has a correlation of .933 with the 6-month Canadian T-bill but of only .405 with the 3-month US Treasury. Thus, the data is supportive of a model with high degree of correlation among

[1] The long-run correlation matrix is similar and is not reported.

cross-sectional units such as our factor model.

3 Analysis of Factors

Our first step in our analysis of the nonstationary properties of our panel is to analyze the behavior of the common factors. This entails estimating the total number of common factors and determining how many of these are nonstationary. We will also attempt to relate the estimated factors to observable variables of interest. The first subsection discusses our methodology, while the second one reports our empirical results.

3.1 Methodology

3.1.1 Estimation of number of factors

One of our main goals is to determine the number of factors in the possibly nonstationary panel model $(1)-(3)$. To this end, we will employ information criteria as suggested by Bai and Ng (2002). These information criteria will be applied to factors estimated by principal components either on residuals (following the Moon and Perron (2004) approach) or on first differences (following Bai and Ng (2004)).

To estimate the true number of factors, K, Bai and Ng (2002) propose to minimize the following criterion functions,

$$PC\left(r\right)=\hat{\sigma}_{e}^{2}\left(r\right)+rG_{nT},$$

$$IC\left(r\right)=\ln\left(\hat{\sigma}_{e}^{2}\left(r\right)\right)+rG_{nT}, \tag{4}$$

where r is the number of factors included in the model, $\hat{\sigma}_{e}^{2}\left(r\right)$ is the variance of the estimated idiosyncratic components, and G_{nT} is a penalty function that depends on the size of the panel. These criteria are similar in spirit to the common AIC and BIC criteria for time series. They involve a trade-off between some measure of fit (as measured by $\hat{\sigma}_{e}^{2}\left(r\right)$) and a function $G_{n,T}$ that acts as penalty for more complex models. The penalty function has to satisfy the conditions (i) $G_{n,T}\to 0$ and (ii)

$\min\{n,T\} G_{n,T} \to \infty$. as both $n$and T go to infinity. As we increase the number of factors, the fit must improve (*i.e.* $\hat{\sigma}_e^2$ goes down), but the penalty term increases. The estimated number of factors is the integer that minimizes the appropriate criterion.

Bai and Ng (2002) and Moon and Perron (2004) demonstrate that estimation of the total number of factors by minimizing either the PC or IC criterion is consistent in the sense that the probability that the estimated number of factors equals the true one approaches one as both n and T become large.

Bai and Ng suggest three specific forms of the penalty function for the PC criterion:

$$\begin{aligned} G_{PC,1,nT} &= \hat{\sigma}_e^2\left(K_{\max}\right)\frac{n+T}{nT}\ln\left(\frac{nT}{n+T}\right), \\ G_{PC,2,nT} &= \hat{\sigma}_e^2\left(K_{\max}\right)\frac{n+T}{nT}\ln\left(\min\{n,T\}\right), \\ G_{PC,3,nT} &= \hat{\sigma}_e^2\left(K_{\max}\right)\left(\frac{\ln\left(\min\{n,T\}\right)}{\min\{n,T\}}\right), \end{aligned} \tag{5}$$

where $\hat{\sigma}_e^2\left(K_{\max}\right)$ is the variance of the idiosyncratic component estimated with the maximum number of factors leading to criteria PC_1, PC_2, and PC_3 respectively, and

$$\begin{aligned} G_{IC,1,nT} &= \frac{n+T}{nT}\ln\left(\frac{nT}{n+T}\right), \\ G_{IC,2,nT} &= \frac{n+T}{nT}\ln\left(\min\{n,T\}\right), \\ G_{IC,3,nT} &= \left(\frac{\ln\left(\min\{n,T\}\right)}{\min\{n,T\}}\right), \end{aligned} \tag{6}$$

leading to the IC_1, IC_2, and IC_3 criteria respectively. Note that, just as in BIC, the IC has the advantage of not requiring the estimation of a scaling factor in the penalty function. We also consider the modified BIC criterion (called BIC_3) :

$$BIC_3\left(r\right) = \hat{\sigma}_e^2\left(r\right) + r\hat{\sigma}_e^2\left(K_{\max}\right)\frac{n+T-r}{nT}\ln\left(nT\right) \tag{7}$$

because simulation evidence suggests that it performs better in selecting the number of factors when $\min(n, T)$ is small (≤ 20) as is often the case in empirical applications. Bai and Ng rejected this criterion because it does not satisfy the required conditions for consistency when either n or T dominates the other one exponentially, but this appears to be a rather unusual case. For small n and T of roughly the same magnitude, this criterion performed best in their simulation among those they considered. For large panels, all three forms of the penalty function are essentially equivalent.

3.1.2 Determining the number of nonstationary factors

When the common component c_{it} in (1) is nonstationary, it may be a linear combination of stationary and nonstationary factors. Once we have determined the total number of factors using the above information criteria on either first differences or residuals, our next task is to determine how many of these factors are nonstationary. For this purpose, we will draw from two approaches. The first one is an extension of the information criteria above. The second is a testing procedure proposed in Bai and Ng (2004) that tests the rank of the long-run covariance matrix of the factors.

Bai (2004) proposed new information criteria to select the number of nonstationary factors in a panel. These are closely related to the information criteria in Bai and Ng (2002) discussed above, but they are applied to the levels of the series rather than the first differences or the residuals. The three criteria are:

$$\begin{aligned}
IPC_1(r) &= \hat{\sigma}_u^2(r) + r\alpha_T \hat{\sigma}_u^2(K_{\max}) \frac{n+T}{nT} \ln\left(\frac{nT}{n+T}\right), \\
IPC_2(r) &= \hat{\sigma}_u^2(r) + r\alpha_T \hat{\sigma}_u^2(K_{\max}) \frac{n+T}{nT} \ln\left(\min\{n,T\}\right), \\
IPC_3(r) &= \hat{\sigma}_u^2(r) + r\alpha_T \hat{\sigma}_u^2(K_{\max}) \frac{n+T-r}{nT} \ln(nT),
\end{aligned} \tag{8}$$

where $\hat{u}_{it} = z_{it} - \hat{d}_{it} - \hat{c}_{it}$ are the estimated idiosyncratic components from level data z_{it} and $\alpha_T = T/\left[4\ln\ln(T)\right]$. Thus, the IC and PC criteria are used to estimate the total number of factors (stationary and nonstationary), while the use of these IPC criteria on levels estimates the number of nonstationary factors. The criterion we will use is the IPC_1 criterion. Bai also suggested the use of BIC on level data to estimate the number of nonstationary factors. This seems to perform better for smaller panels. Note that the consistency result in Bai (2004) requires that the idiosyncratic component be stationary. In the next section, we provide evidence that suggests that this condition is in fact met in our data.

In addition to these information criteria, we will look at the modified Q_c (MQ_c) statistic of Bai and Ng (2004) to determine the number of nonstationary factors in our panels. The statistic tests whether the smallest eigenvalue of the matrix of a first-order VAR is unity and proceeds in a sequential fashion as in the standard Johansen technique. We first start assuming that all $\hat{K}$ factors are nonstationary. If this is the case, all of the eigenvalues from the VAR matrix are unity, and we cannot reject the null hypothesis that the number of nonstationary factors equals the total number of factors. If we can reject this null hypothesis, we move on to tests whether there are $\hat{K}-1$ nonstationary factors and so on. We stop when we cannot reject that the smallest eigenvalue is 1. The limiting distribution of the MQ_c statistic is nonstandard, and critical values are provided in Bai and Ng (2004) for up to 6 factors. Finally, to confirm our results, we will compute the KPSS statistic for testing stationarity on each estimated factor. This procedure has been shown to be valid in Bai and Ng (2005) .

3.1.3 Inference on estimated factors

Recently, Bai (2004) has obtained inferential results for nonstationary factors. In particular, these allow to test whether some observable variable can be argued to represent these factors.

The inherent problem with the estimated factors is that they are only identified up to normalization (they are a basis to some space only). The idea of Bai to circumvent this problem is to first rotate the estimated factors towards the observed series of interest by estimating the linear regression:

$$R_t = \alpha + \delta' F_t + \eta_t. \tag{9}$$

where R_t is some observed series and F_t are the estimated factors. In our results below, we will report the R^2 of this regression for each individual series in the panels and some spreads (in levels and in first differences to prevent against a spurious regression) for one and two factors. Of course, we could also search for observables that are proxied by these factors outside of our panels, but the series in the panels are natural first candidates.

Bai also proposed a procedure for testing whether one of the factors represents the observed series, R_t. The idea is to compare the observed series R_t and the fitted values from (9) , $\hat{R}_t$. The construction of these confidence intervals for $\hat{R}_t$ require the factors to be nonstationary with stationary idiosyncratic errors, otherwise the above regression is a spurious regression (see Phillips (1986)). These distributional results are pointwise (i.e. for each t) which means that if a factor truly represents the observed series R_t, we should see (under independence) $\alpha\%$ of the observations fall outside of a $1-\alpha\%$ confidence interval constructed in this fashion. In consequence, we will also report in our tables below the percentage of observations that fall outside the appropriate 95% confidence interval. If the asymptotic analysis is accurate and one of the included factors in the above regression represents the observed series, we should expect to see entries close to 5% in these columns.

3.2 Empirical results

In this section, we apply our procedures to the analysis of our panel of nominal yields. We first start by estimating the total number of factors. Typically, the term structure literature employs three factors (see Litterman and Scheinkman, 1988). However, the number of factors does not seem to be well estimated by the Bai and Ng information criteria. The IC_1 suggests the presence of 8 factors (the maximum number we allowed), while BIC suggests 7 factors.

Table 2 reports our evidence regarding the number of nonstationary factors. Using the MQ_c statistic, we find a single nonstationary factor, regardless of the maximum number of factors. However, results with information criteria are more fragile. With a maximum of 8 nonstationary factors, the IPC criterion finds 4 nonstationary factors while the BIC finds 3. After setting the maximum number of factors to 4, the IPC finds 3 factors and the BIC finds 2 factors. If we allow only 2 factors, all criteria find a single one as with the MQ_c statistic. Finally, the KPSS statistic rejects stationarity for only one factor. Thus, our results point to the presence of a single nonstationary factor.

To determine whether our estimated factors proxy some variables of interest, we regressed each interest rate on a constant and either the first one or two estimated factors. The first two columns of the table report the R^2 from regression (9) when a single factor is included on the right-hand side. The results from this regression are presented in table 3 for each interest rate in the panel and for some interest rate spreads. The table reveals that all rates are highly correlated with the first factor, with the highest correlation for the 2-year Canadian rate. The next two columns report the R^2 from regression (9) when a second factor is added to the regression. We see a large increase in the R^2 for the spreads, in particular the Canadian term spread. Finally, the last two columns provide the rejection rates of the 95% confidence intervals for the rotated factors using the Bai (2004) methodology. If the estimated factor proxied the corresponding observable variable, we would expect to reject 5% of the time. The information in these columns corroborates the information from the R^2. The lowest rejection rate with one factor is with the 2-year Canadian rate, and the addition of a second factor has most impact on the Canadian term spread. There is much less impact on the US term spread or on the spread between the two countries. Nevertheless, we see that all rejection rates in the table are much above the nominal 5% level. This suggests that the first two factors do not represent one of the rates or spread in our panel.

Figure 1 plots the time series of the 2-year Canadian bond rate against the limits of the 95% confidence interval for the rotated first factor. For illustration we use the case with a single factor, but the picture with two or even three factors is almost identical. We see that the fit is generally pretty good, but that large parts of the series lie outside the bands. If the factor proxied the 2-year Canadian rate, we would see (under independence over time) 5% of the observations outside the limits of the confidence band. Since about 27% of them do, we must conclude that no single interest rate in the panel can proxy for our first estimated factor.

In the empirical literature on the term structure, many models with unobservable factors are used.

In these studies, it is common to find that three factors are necessary to account for the term structure (see Litterman and Scheinkman (1988), Andersen, Benzoni, and Lund (2004) or Diebold and Li (2006) for recent examples), with these three factors usually associated with "level", "slope" and "curvature". It is clear that our first factor can be labelled a level factor that shifts the entire set of yields. Usually, this level factor is proxied by a short rate, but our results suggest that a 1 or 2 year rate provides a better fit. Further evidence that this first factor affects all rates similarly comes from the fact that the loadings are similar for all rates in the panel (results available from the authors upon request).

The second factor, the "slope" or steepness of the yield curve, is usually proxied by a spread between a long rate and a short rate. It is therefore reasonable that the inclusion of a second factor in the regression has a large impact on term spreads, mostly Canadian but also American as evidenced by the much higher R^2 (multiplied by 10 in first differences for both term spreads). Neither of these two factors seems to have much to do with the yield differentials between the two countries however. Figure 2 plots the Canadian term spread and the limits of the 95% confidence bands for the case with two factors. Once again, we see that the overall fit is pretty good, but that the confidence interval does not contain the observed series for many periods, mostly associated with large swings (either up or down) in the spread. Note that there are only two episodes when the Canadian yield curve was inverted (negative spread) for more than one month, in early 1986 and late 1989-early 1990. The confidence interval appears rather narrow, and this could be due to the fact that we have treated this second factor as nonstationary in the construction. Our results above suggest the strong possibility that this second "slope" factor is in fact stationary. This would invalidate the reported intervals. It is also interesting to note that the loadings on this second decrease monotonically with the term to maturity of Canadian government bonds. Thus, the second factor affects the relative magnitude of short versus long Canadian bonds. The pattern among loadings is not as clear among US Treasury bonds.

There has been much recent literature on the links between term structure models and macroeconomic variables. For example, Diebold, Rudebusch, and Aruoba (2006) try to compare their three-factor term structure model to macro variables. They find that their level factor is highly correlated with inflation. They interpret this evidence as being consistent with the Fisher equation of a link between nominal yields and inflationary expectations. Their second factor, on the other hand, is closely linked to levels of economic activity. They find a high correlation between their second factor and a measure of capacity utilization. This correlation is also the source of the use of the term spread to forecast business cycle fluctuations as in Estrella and Mishkin (1998). The yield curve tends to be downward-sloping (short rates are higher than longer rates) prior or at the beginning of recessions. Our second factor is therefore likely to be related to the business cycle.

4 Analysis of Idiosyncratic Components

Our second step in our analysis of the nonstationary properties of our panel is to look at the idiosyncratic components, u_{it} in our decomposition (1). The analysis of the nonstationarity of the idiosyncratic component is much more developed than the analysis of the stationarity properties of factors and proceeds in two steps: in the first step, some estimate of the idiosyncratic components is obtained, while in a second step, a panel unit root test is applied. The estimation of the idiosyncratic component is equivalent to obtaining estimates of the deterministic and common components.

In the previous section, we used principal components on first differences to obtain estimates of the factors. The idiosyncratic components could then be obtained as the residuals after removing the estimated common components from the observed data. We chose this method because it allowed us to use inferential procedures on the estimated factors. Other available methods for estimating the common and idiosyncractic components do not provide inferential procedures for factors. These methods include principal components on residuals as in Moon and Perron (2004), the moment-based orthogonalization method of Phillips and Sul (2003), and the cross-sectional average of Choi (2006*a*) and Pesaran (2006). Both the Bai and Ng and the Moon and Perron methods estimate general dynamic factor models with an unknown number, K, of factors. On the other hand, Phillips and Sul's method is designed for a model with a single i.i.d. factor, while Choi uses an error-component model (which is a single factor model with homogeneous factor loadings).

The second step in our analysis consists in testing the estimated idiosyncratic components for a unit root. We will use a total of seven unit root tests in this paper. Each assumes that cross-sectional correlation is captured via a factor model. This restricts us to focus on tests that require both n and T to diverge since a large n is necessary for consistent estimation of the factors. Other methods of dealing with cross-sectional correlation do not face this restriction and may be better suited for smaller panels. We will not review all the tests in detail since recent surveys are available in Choi (2006b), Hurlin and Mignon (2004) and Breitung and Pesaran (2005).

The null hypothesis of interest is that all idiosyncratic components are nonstationary:

$$H_0 : \rho_i = 1 \text{ for all } i = 1, \ldots, n,$$

whereas the alternative hypothesis takes the form:

$$H_A : \rho_i < 1 \text{ for some } i,$$

where ρ_i is the largest autoregressive root in the time series of individual i.

For the tests to be consistent, it is necessary that a positive fraction of the units fall under the alternative (stationary) hypothesis, *i.e.* $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 1(\rho_i < 1) > 0$ where $1(\cdot)$ is the indicator function.

Early panel unit root tests (e.g Levin et al. (2002), Im et al. (2003), and Maddala and Wu (1999) or Choi (2001)) assumed independence of observed data across individual units. This assumption is clearly unrealistic in empirical settings as suggested by the large correlations reported in table 1. However, once factors have been extracted from the data, similar methods can be applied to *the idiosyncratic components*. In other words, once we have estimated idiosyncratic components, we could apply the pooling approach of Levin et al. (2002), the averaging approach of Im et al. (2003), or the p-value combination approach of Maddala and Wu or Choi (2001) to test the joint null hypothesis of nonstationary idiosyncratic components.

The seven tests we consider are those of Moon and Perron (2004) , Bai and Ng (2004) , Phillips and Sul (2003) , Choi (2006*a*) , Pesaran (2006) , a panel version of the Sargan and Bhargava (1983) test developed in Moon and Perron (2005), and a common point optimal test proposed in Moon, Perron, and Phillips (2005) with $c = 1$. These tests vary according to the way they eliminate the factors and the way they aggregate the individual information. We will first use the defactoring method of Moon and Perron (2004) before applying these last two tests.

The CIPS test developed by Pesaran (2006) is slightly different in nature. Pesaran showed that by augmenting the usual ADF regression with the first difference and the first lag of the cross-sectional mean, one can account for the cross-sectional dependence arising through a single stationary factor. Thus, no direct estimation of the idiosyncratic component is needed in this approach. This can be beneficial for small panels where estimation of factors is difficult.

As with the factors, we will perform stationarity tests on the estimated idiosyncratic components. In order to do so, it is first necessary to project the estimated idiosyncractic components in the space orthogonal to a constant and the nonstationary factors. The validity of this approach has been shown in Bai and Ng (2005) . Note however that we can only perform individuals KPSS tests on the estimated idiosyncratic components. Pooling is not possible due to the presence of nonstationary factors as the nonstationarity will be transmitted to the residuals in a non-vanishing way under the null hypothesis of stationarity.

Other tests for panel unit roots with cross-sectional dependence that do not rely on a factor structure are available. Recent tests of this kind that have been proposed by Chang (2002) , Breitung and Das (2005) , Shin and Kang (2004) , and Choi and Chue (2005) , while an earlier one was proposed by Taylor and Sarno (1998) . These tests allow for general cross-sectional dependence of the error terms and, typically, do not need to let the number of cross-sections, n, diverge since no estimation of the factors is necessary.

4.1 Empirical results

Our unit root test results are presented in table 4. For the tests that require the estimation of the number of factors (Moon and Perron, Bai and Ng, and the two optimal tests combined with the Moon and Perron defactoring procedure), we report results as the number of factors varies between 1 and 8 since the estimated idiosyncratic components depends on the assumed number of factors. We report results in this way because the number of cross-sections does not appear to be sufficient to get a good estimate of the number of factors as we saw above. For the tests that are based on individual ADF tests and the CIPS test, we choose the number of lagged first differences to be included by AIC in all cases with a maximum of 12 lags. When needed, long-run variances and covariances are computed with a quadratic spectral kernel estimate and with the bandwidth selected following the rule of Andrews (1991) and with prewhitening.

Fortunately, the results from the application of panel unit root tests are clear and do not depend much on the estimated number of factors. With the exception of the Moon-Perron-Phillips and Sargan-Bhargava tests with 1 or 2 factors and the Bai and Ng test with 2, 4, 5, and 6 factors, we reject the null hypothesis of a unit root in all idiosyncratic components.

The results from the KPSS tests confirm these results. The last row of table 4 includes the number of idiosyncratic series for which we cannot reject stationarity at the 5% level. Regardless of the number of factors we allow, we cannot reject stationarity for at least 23 series. Moreover, we reject stationarity for a total of only 6 out of the 200 tests that we perform. As discussed above, it is not possible to aggregate these individual tests in order to control the overall rejection probability. However, these results are indicative and further support our claim that most, if not all, idiosyncratic components are stationary.

The presence of nonstationary factor(s) and stationary idiosyncratic components means that the nominal yields in our panel are cointegrated. Similar conclusions (nonstationary factors and stationary idiosyncratic components) are obtained when applying our approach to the yields of each country separately. This corroborates many empirical results supporting cointegration in the term structure dating back to Campbell and Shiller (1987) and Evans and Lewis (1994) and provides support for models of cointegration in the term structure such as that of Carstensen (2003).

5 Conclusion

This paper has analyzed a panel of interest rates with different maturities and risk characteristics. Our application of recently-developed methods for nonstationary panels where the units are correlated through a factor structure suggests that a single nonstationary factor is sufficient to model these data. However, the number of stationary factors needed to model the cross-sectional correlation is not well identified. Since we can reject the nonstationarity of all idiosyncratic components, these results suggest that interest rates are cointegrated. The dominant factor in the interest rate panel is a level factor that is highly correlated with all rates and could be the result of inflationary expectations. The second factor has an interpretation as a slope factor, that is the differential between a long rate and a short rate since it affects short and long rates differently and might be a measure of the business cycle.

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6 Appendix: Simulation evidence

In this appendix, we present limited simulation evidence to document the behavior of the tests for the nonstationarity of the idiosyncratic components considered in this paper. Other (more comprehensive) comparative simulation evidence can be found in Gutierrez (2006) and Gengenbach et al. (2005).

The data-generating process is a slightly modified version of the data-generating process of Moon and Perron (2004). The modification greatly increases the level of cross-sectional dependence by replacing the $N(0,1)$ factor loadings with $U[0,1]$ factor loadings.

The data-generating process is given by equations (2):

$$\begin{aligned}
z_{it} &= \alpha_i + z_{it}^0 \\
z_{it}^0 &= \rho_i z_{it-1}^0 + y_{it} \\
z_{i0}^0 &= 0
\end{aligned}$$

with a single factor structure for the error terms as in equation (3):

$$y_{it} = \tau \beta_i f_t + e_{it}.$$

All shocks are assumed i.i.d. standard normal:

$$(f_{tj}, e_{it}) \sim iidN(0, I_2)$$

while the factor loadings are $\beta_{ij} \sim iidU[0,1]$, and the deterministic components are heterogeneous, $\alpha_i \sim N(0,1)$.

Size is considered for $\rho_i = 1$ for all i. We fix the alternative to be uniform with a mean of 0.99, $\rho_i \sim U[0.98, 1]$. Finally, we consider two values of the parameter controlling the relative importance of common versus idiosyncratic shocks τ, 1 and 3. We choose three values for n (n = 10, 20, and

100) and two values for T ($T = 100$ and 300). We estimate long-run variances even though it is not necessary to do so (since shocks are independent over time). When necessary for the factor extraction method employed by a given test, the number of factors is estimated by the IC_1 criterion of Bai and Ng (2002) with a maximum of 8 factors. Note that this design does not satisfy the assumptions of the Choi test since factor loadings are heterogeneous. We therefore expect to see large size distortions for this test.

Note that our design is also limited since the factors and idiosyncratic components are restricted to have the same order of integration. Under the null hypothesis, Δz_{it}^0 is made up of a stationary common component and a stationary idiosyncratic shock so that both components are nonstationary in levels. Under the alternative hypothesis, z_{it}^0 is the sum of two stationary components.

The results are presented in tables A1 (size), A2 (power), and A3 (size-adjusted power). The first thing to notice is that all tests that require the estimation of the number of factors (MPP, Moon and Perron, Sargan and Bhargava, and Bai and Ng) have severe size distortions for the smallest choice of n. This is due to overestimation of the number of factors as reported in Moon and Perron (2004). For $n = 10$, the IC_1 criterion tends to choose the maximum number of factors allowed (8). For $n \geq 20$, this problem disappears since the estimated number of factors is almost always equal to the true one. The $CIPS$ and Phillips and Sul tests are not affected by this problem since the first one does not estimate factors directly, while the second one imposes (correctly in this case) the presence of a single factor. As expected, the Choi test has large size distortions. These are reduced when T is large and τ is small. The Moon and Perron and Bai and Ng tests tend to overreject for $n \geq 20$. The MPP tends to slightly underreject for $c = 1$ and overreject for $c = 2$. Nonetheless, it is clear that the combination of the Moon and Perron (2004) defactoring procedure with this common point optimal tests leads to reasonable size control.

The size-adjusted power results suggest that the MPP, Moon and Perron, and Sargan-Bhargava tests dominate and are pretty much equivalent. The Bai and Ng test is next, followed by Phillips and Sul, and finally CIPS. Table A3 also reproduces the result in Moon, Perron, and Phillips (2005) that the power of their test is not sensitive to the choice of c (except maybe for $n = 10$). Finally, the table also shows that, as expected, power goes down with the degree of cross-sectional correlation, *i.e.* power is lower for $\tau = 3$ than for $\tau = 1$.

In conclusion, among the tests considered in this study, the CIPS test of Pesaran (2006) is best at controlling size, in particular for smaller panels. It is also the easiest to compute. However, its power for alternatives that are close to the unit root null hypothesis is quite low. It should therefore be emphasized in situations with small panels and alternatives of interest that are not too close to unity. In other cases, tests that estimate the factor model are called for. In particular, if cointegration is suspected, the Bai and Ng procedure must be given precedence since it is the only valid procedure in such cases.

Table 1. Short-run correlation matrix
First differences of monthly Canadian and U.S. interest rates
1985-2004

		Canadian rates														U.S. rates									
		1 mth	3 mths	6 mths	1 yr	2 yrs	3 yrs	5 yrs	7 yrs	10 yrs	1-m CP	3-m CP	1-m BA	Long corp.	Mid corp.	3 mths	6 mths	1 yr	2 yrs	3 yrs	5 yrs	7 yrs	10 yrs	1 m CP	AAA
	3 months	.872																							
	6 months	.697	.933																						
	1 year	.583	.851	.966																					
	2 years	.466	.704	.834	.910																				
	3 years	.414	.649	.783	.862	.962																			
Can.	5 years	.359	.575	.716	.802	.924	.961																		
rates	7 years	.334	.545	.690	.778	.889	.928	.977																	
	10 years	.276	.474	.621	.710	.830	.883	.955	.976																
	1 m com. paper	.948	.910	.755	.644	.509	.456	.392	.363	.299															
	3-m com. paper	.863	.979	.924	.848	.698	.642	.567	.535	.466	.928														
	1-m Bank. acc.	.952	.909	.761	.651	.517	.469	.404	.376	.313	.993	.922													
	Long corporate	.233	.440	.591	.667	.757	.815	.885	.911	.940	.266	.440	.277												
	Mid corporate	.292	.514	.670	.759	.860	.900	.948	.954	.954	.322	.513	.333	.953											
	3 months	.323	.405	.418	.426	.401	.394	.365	.326	.310	.325	.380	.340	.237	.300										
	6 months	.278	.372	.404	.426	.431	.434	.420	.380	.365	.306	.362	.315	.294	.361	.937									
	1 year	.248	.359	.414	.459	.500	.501	.504	.473	.461	.279	.347	.289	.365	.429	.856	.953								
	2 years	.193	.304	.373	.436	.523	.524	.548	.526	.515	.222	.295	.228	.408	.476	.732	.863	.952							
U.S.	3 years	.162	.267	.342	.411	.512	.520	.560	.546	.543	.186	.260	.191	.436	.494	.663	.807	.913	.988						
rates	5 years	.147	.242	.316	.388	.492	.506	.559	.550	.556	.169	.239	.171	.456	.502	.598	.744	.857	.958	.984					
	7 years	.137	.228	.304	.377	.484	.511	.574	.574	.590	.158	.226	.160	.499	.527	.543	.694	.811	.926	.963	.989				
	10 years	.122	.205	.276	.347	.453	.485	.556	.559	.584	.144	.206	.146	.501	.516	.500	.650	.766	.886	.930	.970	.989			
	1 m com. paper	.165	.309	.354	.360	.304	.308	.258	.229	.215	.254	.311	.264	.215	.244	.661	.698	.660	.528	.465	.392	.361	.319		
	AAA	.100	.153	.214	.274	.379	.417	.484	.489	.524	.121	.155	.122	.499	.481	.412	.570	.663	.769	.815	.860	.893	.906	.309	
	BAA	.108	.148	.193	.241	.346	.380	.444	.455	.486	.134	.152	.136	.479	.456	.365	.525	.615	.731	.775	.822	.855	.870	.299	.942

Table 2. Estimated number of nonstationary factors

max =	1	2	3	4	5	6	7	8
MQ_c	1	1	1	1	1	1	1	1
IPC	1	1	2	3	3	4	4	4
BIC	1	1	1	2	3	3	3	3

The table reports the estimated number of nonstationary factors with the maximum number of factors in the first row.

Table 3. R^2 of regression each interest rate on factors
Sample of 25 monthly interest rates
1985-2004

		R^2 (%)				$Pr\,(z_{it} \notin CI)$	
		$K=1$		$K=2$		$K=1$	$K=2$
		Levels	First differences	Levels	First differences		
Canadian rates	1-month T-bill	91.4	42.3	98.6	82.4	63.8	48.3
	3-month T-bill	93.5	67.2	99.4	96.9	57.8	22.0
	6-month T-bill	95.5	76.0	99.4	89.1	44.8	37.9
	1 year	97.0	79.3	98.9	83.6	32.8	53.9
	2 years	98.1	79.8	98.2	79.9	27.2	54.7
	3 years	97.8	78.3	97.9	79.8	33.6	53.9
	5 years	96.2	76.2	97.1	82.3	51.7	67.2
	7 years	93.9	72.5	95.8	80.3	58.2	70.3
	10 years	91.3	66.0	94.5	78.7	63.4	69.8
	1-month commercial paper	91.8	48.5	99.1	89.5	60.3	32.3
	3-month commercial paper	93.5	66.7	99.6	96.9	56.0	14.7
	1-month Bankers' acceptances	91.9	49.5	99.2	89.7	60.8	32.8
	Long-term corporate (Scotia)	85.9	56.2	88.2	67.9	74.1	78.5
	Mid-term corporate (Scotia)	91.8	67.1	92.6	75.8	65.5	82.8
US rates	3-month Treasury	78.6	33.7	78.7	34.4	82.3	91.8
	6-month Treasury	79.8	40.2	79.8	45.0	80.6	90.5
	1-year Treasury	81.4	47.1	82.1	57.9	84.1	88.8
	2-year Treasury	84.6	47.9	87.5	69.0	75.9	81.5
	3-year Treasury	86.6	46.4	91.3	73.6	73.7	71.6
	5-year Treasury	88.5	44.6	96.2	76.0	72.8	43.5
	7-year Treasury	88.4	44.2	97.7	79.3	65.5	25.0
	10-year Treasury	87.6	41.0	98.3	77.6	67.2	18.1
	1-month commercial paper	77.5	20.2	77.8	20.4	85.8	93.5
	Moody's AAA	85.2	32.6	96.1	69.2	61.2	37.1
	Moody's BAA	81.3	29.6	91.3	63.1	65.5	61.6
spreads	10y - 3m Canadian	42.7	8.0	93.9	84.1	92.2	37.5
	3m US - 3m Canadian	42.8	42.7	57.6	77.1	94.0	87.9
	10y - 3m US	2.8	3.0	32.3	36.9	100.0	85.3

Table 4 Results of Unit Root Tests on idiosyncratic components
Sample of 25 monthly Canadian and US interest rates
January 1985- April 2004

$K =$	1	2	3	4	5	6	7	8	$\hat{K}_{IC_1}$	$\hat{K}_{BIC_3}$
Moon-Perron	-6.496*	-5.923*	-3.807*	-4.622*	-4.261*	-4.678*	-4.703*	-4.589*	8	7
Bai-Ng	-2.217*	0.912	-2.790*	-1.214	-1.124	-0.965	-5.190*	-7.196*	8	7
Phillips-Sul (Z)				-5.611*						
Choi (Z)				-7.914*						
Pesaran ($CIPS$)				–3.059*						
Sargan-Bhargava	-1.263	-1.489	-2.685*	-2.352*	-1.942*	-2.604*	-2.406*	-3.671*	8	7
Moon-Perron-Phillips ($c = 1$)	-792	-.574	-2.637*	-1.934*	-2.125*	-2.585*	*2.451*	-4.115*	8	7

Note: an asterisk denotes a significant statistic at the 5% level

Table A1. Size of tests

DGP: $z_{it} = \alpha_{i0} + z_{it}^0$

$z_{it}^0 = z_{it-1}^0 + \tau\beta_i f_t + e_{it}$

$\alpha_{i0}, f_t, e_{it} \sim iidN(0,1)$

$\beta_i \sim iidU[0,1]$

	n	T	MPP $c=1$	MPP $c=2$	Moon-Perron	CIPS	Sargan	Bai-Ng	Choi	Phillips-Sul
	10	100	30.8	35.4	29.5	5.6	23.0	17.5	16.9	5.3
	20	100	4.3	6.0	7.4	5.6	2.9	7.0	24.5	5.9
	100	100	4.9	5.9	7.0	6.1	3.9	7.1	69.7	6.4
$\tau=1$	10	300	30.3	34.5	29.8	4.9	24.5	16.4	6.2	5.1
	20	300	4.4	6.2	7.4	5.2	3.3	6.7	5.9	5.6
	100	300	4.7	5.3	6.0	5.7	3.7	5.7	7.5	6.5
	10	100	40.1	45.0	40.6	6.0	33.1	17.7	23.5	5.3
	20	100	4.5	6.4	8.2	5.4	2.4	6.7	35.0	6.1
	100	100	5.0	5.6	9.2	5.8	3.8	7.0	64.3	6.8
$\tau=3$	10	300	35,5	40.7	35.0	5.4	29.3	17.3	9.4	5.3
	20	300	4.0	5.9	7.2	5,1	2.8	6.3	13.4	5.0
	100	300	4.4	5.0	6.4	5.7	4.0	6.6	33.5	6.9

Note: Each entry represents the percentage of replications in which the null hypothesis of a unit root is rejected for the appropriate 5% test with the number of factors estimated using the IC_1information criterion suggested by Bai and Ng (2002) using the asymptotic critical values. The number of replications is 5000.

Table A2. Power of tests against linear alternative

DGP: $z_{it} = \alpha_{i0} + z_{it}^0$

$z_{it}^0 = \rho_i z_{it-1}^0 + \tau\beta_i f_t + e_{it}$

$\alpha_{i0}, f_t, e_{it} \sim iidN(0,1)$

$\beta_i \sim iidU[0,1]$

$\rho_i \sim U[0.98, 1]$

	n	T	MPP $c=1$	MPP $c=2$	Moon-Perron	CIPS	Sargan	Bai-Ng	Choi	Phillips-Sul
	10	100	51.1	57.3	51.7	8.0	39.8	25.5	59.6	11.6
	20	100	56.9	64.9	69.9	8.9	42.2	44.1	87.4	17.9
	100	100	99.6	99.7	99.8	11.0	99.1	94.9	99.8	50.9
$\tau=1$	10	300	74.3	80.0	76.6	18.0	67.8	48.9	88.0	28.9
	20	300	95.8	97.1	97.6	25.8	93.7	97.1	98.7	52.3
	100	300	100.0	100.0	100.0	45.3	100.0	100.0	99.9	97.8
	10	100	57.9	63.8	59.8	7.0	48.6	24.1	56.7	8.8
	20	100	45.3	53.2	60.3	7.5	33.0	36.4	74.8	13.9
	100	100	89.7	90.6	92.1	7.2	88.6	82.4	90.6	36.3
$\tau=3$	10	300	72.9	78.7	75.5	16.8	66.5	42.2	75.0	20.0
	20	300	78.8	81.3	84.1	20.9	75.9	83.5	87.8	31.2
	100	300	93.1	93.4	93.7	36.0	93.8	97.9	96.3	69.2

Note: See table A1.

Table A3. Size-adjusted power of tests against linear alternative

$$\text{DGP: } z_{it} = \alpha_{i0} + z_{it}^0$$
$$z_{it}^0 = \rho_i z_{it-1}^0 + \tau \beta_i f_t + e_{it}$$
$$\alpha_{i0}, f_t, e_{it} \sim iidN(0,1)$$
$$\beta_i \sim iidU[0,1]$$
$$\rho_i \sim U[0.98,1]$$

	n	T	MPP $c=1$	MPP $c=2$	Moon-Perron	CIPS	Sargan	Bai-Ng	Choi	Phillips-Sul
	10	100	10.3	10.4	11.3	7.4	10.0	8.4	30.3	10.2
	20	100	60.6	62.0	62.2	8.1	53.2	36.2	53.5	16.3
	100	100	99.6	99.7	99.7	9.3	99.3	93.0	96.6	45.9
$\tau=1$	10	300	22.3	23.7	27.3	18.8	23.9	20.4	85.8	28.6
	20	300	96.2	96.6	96.5	25.0	95.8	96.0	98.4	49.8
	100	300	100.0	100.0	100.0	42.2	100.0	100.0	99.9	96.9
	10	100	8.6	8.4	10.6	6.1	8.9	7.0	24.3	8.2
	20	100	47.7	49.0	51.1	7.0	43.2	31.0	33.0	11.7
	100	100	89.7	89.9	89.8	6.3	89.8	77.0	51.2	30.9
$\tau=3$	10	300	19.1	20.8	22.4	15.9	20.0	16.0	66.0	19.0
	20	300	80.0	80.3	81.6	20.5	80.1	80.7	79.0	31.2
	100	300	93.4	93.4	93.4	32.3	94.1	97.4	89.7	65.8

Note: Each entry represents the percentage of replications in which the null hypothesis of a unit root is rejected for the appropriate 5% test using the empirical critical values. The number of replications is 5000.

Figure 1. Time plot of 2-year Canadian bond (solid line) vs. 95% CI for rotated factor (dashed lines) - single factor

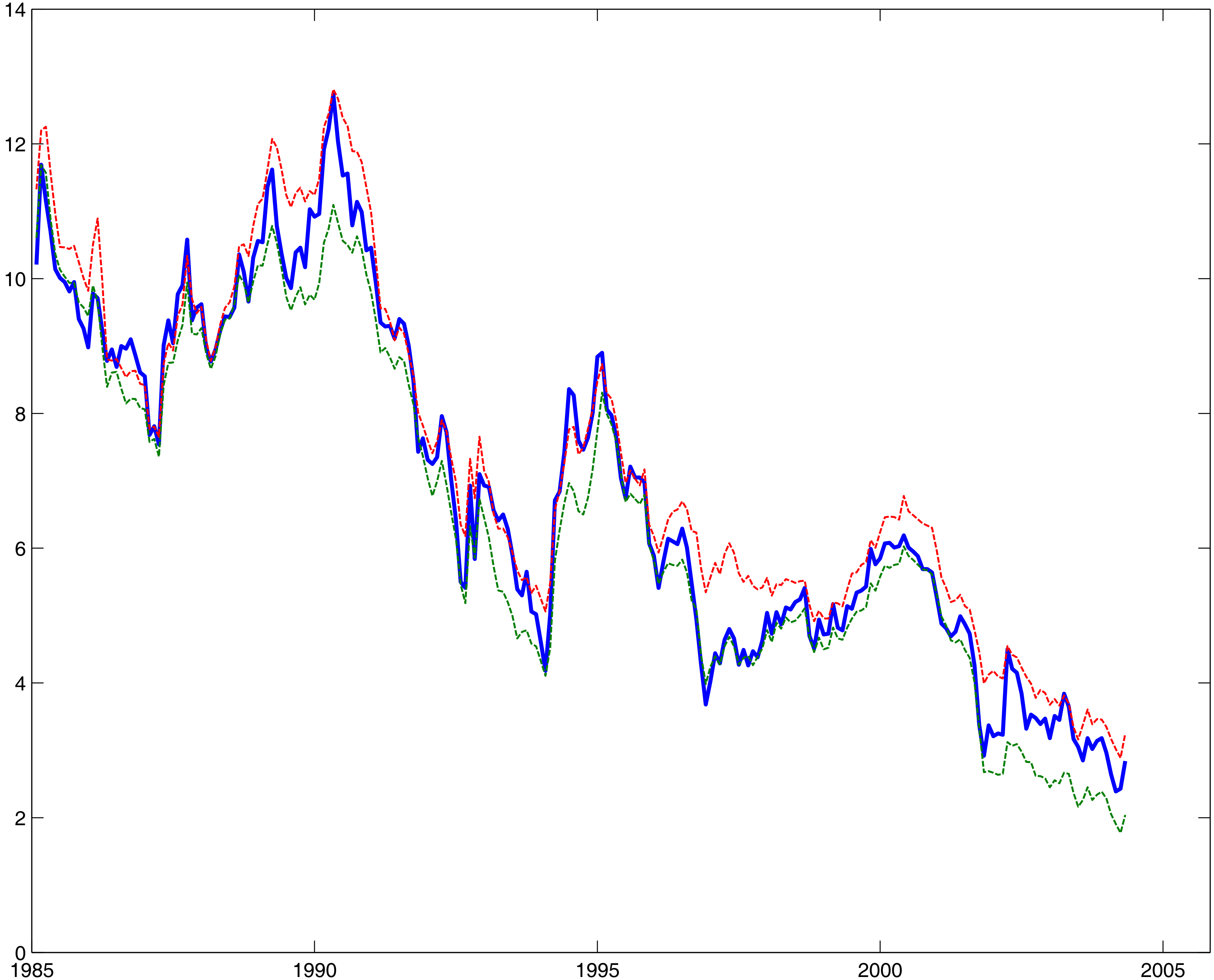

Figure 2. Time plot of Canadian spread (solid line) vs. 95% CI for rotated factors (dashed lines) - 2 factors